A Model of Tax Evasion with Heterogeneous Firms

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ABSTRACT

This paper develops a static industry model of tax evasion with heterogeneous producers to look at the effects of tax rates, penalty costs, and the probability of getting caught on the size of tax evasion. I assume that the probability of tax detection depends on the level of production. If the firm’s production level is smaller than a predetermined level of output, then the firm is not going to be caught for tax evasion. Once the firm’s production exceeds the output threshold, the tax evading firm will be monitored and punished by the tax administration if caught. Under this assumption, I show that the firms which decide to go underground are small firms whereas large firms fulfill their tax obligations. I calibrate the model for the case of Russia for the year of 1998. To estimate the model, I use Goskomstat data, the World Bank data, and the micro-level data on 5537 Russian firms provided by ZAO “Russian investment corporation”. Using the firm's level data, I construct the size distribution of Russian firms that allows me to visually identify tax evading firms. I also invent a method that allows me to calibrate an effective tax rate. The model predicts that the higher tax rates, as well as low penalty costs and low probability of detection, encourage the concealment of activity. A larger informal sector is also generated by reducing penalty costs or reducing the lower bound of output at which the government identifies tax evaders. An increase in the probability of detection and decrease in tax rates lead to a larger reduction in the size of tax evasion than a simultaneous increase in the probability of detection and increase in penalty costs.

Keywords: Tax evasion, Monopolistic competition, Heterogeneous firms, Russia
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1 Introduction

Benjamin Franklin (1706-90) once wrote, “In this world nothing is certain but death and taxes”. This fatalistic and sardonic aphorism is used to say that there were two unavoidable things in life: death and taxes. It seems that in today’s world only one of these things is certain and unavoidable. A high tax burden, high intensity of government regulation, high incidence of bribery, and a weak rule of law drive a number of individuals and firms into the shadow economy.

The methods by which individuals and firms reduce their tax liabilities take a variety of legal and illegal forms. Such methods are broadly classified as avoidance and evasion. Tax avoidance is any legal utilization of existing tax loopholes to lower taxes, such as worker substitution between wage and nonwage compensation. Tax evasion is any illegal activity whereby individuals or firms deliberately conceal their true income to reduce tax liabilities. It includes dishonest tax reporting such as declaring less income than actually earned or overstating deductions. In this study, any legal economic activities which are not taxed or registered but would generally be taxable were they reported to the tax authorities will be viewed as tax evasion.

Tax evasion and corruption exist to some degree everywhere and is particularly acute in the developing and transition economies. Schneider (2004) makes an attempt to estimate the size of the shadow economy across 145 countries around the world (including transition, developing and developed) over the period 1999 to 2003. He finds that in 2002-2003 the smallest underground sector was observed in the United States, Switzerland, and Austria and varied between 8.4 – 10.0 percent of official GDP. The central European countries (the Netherlands, France, Germany, and Great Britain) - countries with comparatively high tax morale and high expected punishment - had the underground sector between 12-17 percent of official GDP. The Netherlands, Germany, Denmark, Belgium and Italy have shadow economies nearly one-third as large as officially measured GDP. The underground sector in transition countries like Russia, Moldova, Belarus, Ukraine, and Georgia – countries with a high tax burden and a high level of corruption – have a shadow economy over one-half the size of officially recorded

1 Schneider and Enste (2000, p. 78) define the shadow economy as “all economic activities that contribute
GDP. Analyzing the growth of the size of shadow economy across countries, Schneider concludes that in developed countries the shadow economy tends to shrink over time while in developing and transition economies it gets larger and larger each year. It is thus clear that tax evasion is an issue of first importance in underdeveloped countries.

The reason for studying tax evasion is self-evident. First, tax evasion is important as it imposes a considerable burden on the government budget. Loyaza (1996) argues that an increase in the shadow economy leads to an increase in tax revenue loss and therefore causes serious damage to the public sector, threatening its capacity to finance public services and goods. Second, it places a disproportionate share of the tax burden on honest taxpayers, and thereby distorts economic decision-making. Therefore, the need for studying tax evasion seems to be of special significance these days.

There exists a number of theoretical, experimental, and empirical papers regarding tax evasion by individuals, however, research on tax evasion by businesses is surprisingly modest. We know little about how the level of noncompliance varies by firm size. The reasons are simple. First, there is a lack of data and credible methods to estimate tax evasion on the micro-level. Second, policies that identify evaders vary across countries. Tax authorities might believe that the rich are most likely to evade taxes. Or they might base their policy on some statistical hypothesis that in the absence of any information about actual production, a firm with a low output is more likely to be an evader. Here I would like to find what types of firms, efficient or inefficient, easily get away with cheating. The need for a model which might capture the firm’s tax compliance decision is indeed unquestionable.

In this paper I propose a static heterogeneous-firm general equilibrium model of tax evasion where firms differ in their productivities as in Melitz (2003) industry model. I assume that that the probability of tax detection depends on the level of production with large firms facing a higher probability of detection. This results in the situation where small firms evade taxes whereas large firms pay taxes.

I calibrate my model to the case of Russia where the problem of tax evasion is particularly acute. There is a widespread perception among taxpayers in Russia that they do not owe the government anything because the government doesn’t do anything for them and so they evade taxes. Nearly half of the taxes owed in Russia never get paid
(Anon, 1998). The Russian government is keen to maximize its tax revenue from all sources, as its budget is under extreme pressure. In 1998, tax chief Boris Fyodorov announced that he would target the rich individuals, adjusting tax focus on large enterprises such as the oil companies. The government tries to squeeze more revenue from the large domestic companies by removing all the tax privileges previously available under Russian law. Targeting the high profile individuals, the Russian government was aiming to send a message to all Russians who consider evading taxes and encourage them to start paying taxes.

I use Goskomstat data, the World Bank data and the Russian firm-level data provided by ZAO “Russian investment corporation” to calibrate the parameters of my model. The firm data has statistical budget reports over the period of 1997 until 1999 for 49,829 Russian firms. I choose the year of 1998 and have 5537 firms in my sample. I construct a method to identify tax evaders/tax payers and determine the effective payroll tax rate. I do a series of comparative statics experiments to quantitatively investigate the firms’ choice to pay or not pay taxes in response to changes in the tax burden, tax evasion penalties, and probability of detection.

The results show that a decrease in the tax rate will always increase the reported income and decrease tax evasion. An increase in the penalty rate also induces tax compliance having all other parameters fixed. A small increase in the probability of detection at first increases a fraction of tax evaders who fix their output at a lower predetermined level of output to evade high payroll costs, but as it becomes larger, fewer firms choose to go underground. A large percentage change in the probability of detection and a low tax rate help to fight tax evasion.

The paper is divided as follows: Section 2 reviews the literature on tax evasion; Section 3 describes the data; Section 4 describes the model of tax evasion in which small-sized firms evade taxes, the agents’ optimal behavior and defines the competitive equilibrium allocations; Section 5 shows how to calibrate the model, discusses the case of Russia in 1998 and solves the model numerically. Finally, Section 6 provides some concluding remarks and policy implications.
2 Theoretical Literature

In this section, I will review the classic theoretical literature and empirical studies on tax evasion and discuss the main implications and findings. I will discuss static models, the main methods to estimate the size of tax evasion and identify the key determinants of tax evasion behavior.

In the early 1900s, tax evasion as a topic for theoretical investigation was in fact neglected and primarily considered by sociologists and anthropologists. Only in the 1950s and 1960s did economists start to show interest to the activities carried out outside the formal framework of the economy. Many have been trying to analyze the behavior of economic agents who engage in illegal activities and the incentives encouraging them to do so by developing formal models and applying them to a variety of socioeconomic problems, such as tax evasion. Yet there persist contradictions and inconsistent outcomes among research studies about tax evasion estimation procedures and the use of estimates in economic analysis and policy.

The classic paper on tax evasion was written by Allingham and Sandmo (1972), hereafter A. and S. They assume that the taxpayer is faced with the decision of whether to pay taxes or to evade taxes. He chooses an amount of income to declare to maximize the von Neumann-Morgenstern (VNM) expected utility function:

\[
E[U] = (1 - p)U(W - \theta X) + pU(W - \theta X - F(W - X)),
\]

where \(W\) is the actual income\(^2\), \(\theta\) is the income tax rate, \(p\) is the probability of detection, \(X\) is the amount of declared income, \(F\) is the fine rate levied on the amount of concealed income that equals \(W - X\). The comparative static analysis of A. and S.’s model implies that an increase in the penalty rate \(F\) and/or an increase in the probability of detection \(p\) will always increase the reported income \(X\) and therefore decrease tax evasion. However, an increase in the tax rate has an ambiguous effect on the incentives to cheat due to the competing income and substitution effects. On one hand, an increased tax rate has a positive substitution effect since higher tax rate means greater marginal benefit of cheating, and this leads to more evasion. On the other hand, an increased tax rate has a

\(^2\) Actual income is exogenously given and known by the taxpayer but not by the tax collector (Allingham and Sandmo (1972, p. 324))
negative income effect since taxpayers feel less wealthy. Therefore, provided decreasing absolute risk aversion, this effect alone would reduce evasion. Both the substitution and income effects compete against each other and thus it is impossible to say a priori whether higher taxes encourage or discourage dishonesty.

Kolm (1973) described the income tax evasion problem by setting up two separate problems: the taxpayer’s problem and the government’s problem. The taxpayer’s problem is similar to the one described by A. and S. with the only difference that here the utility of the taxpayer is represented by the expected utility from both private goods and public goods:

\[
S = (1 - p)U(W - \theta X) + pU(W - \theta X - F(W - X)) + V(T),
\]

where \( W \) is the actual income, \( \theta \) is the income tax rate, \( p \) is the probability of detecting fraud, \( X \) is the amount of declared income, \( F \) is the penalty rate on unreported income, and \( V(T) \) is the utility of public goods, where \( T \) is the total tax yield used to produce public goods.

He assumes that citizens cannot affect the provision of public goods, so \( T \) is given to a single citizen. Hence, the public goods-driven utility can be ignored in the decision-making. So the taxpayer’s problem looks exactly as in A. and S.’s model: Given the income tax rate, \( \theta \), the penalty rate, \( F \), and the probability of detection, \( p \), the taxpayer chooses \( X \) so as to maximize his expected utility \( EU \) of \( S \):

\[
EU = (1 - p)U(W - \theta X) + pU(W - \theta X - F(W - X)).
\]

Given the optimal taxpayer’s decision, the government chooses parameters \( \theta \), \( p \), and \( F \) to maximize \( S \). Assuming that the taxpayers are identical in their preferences, \( S \) multiplied by the number of citizens gives the sum of the citizen’s utilities. By separately solving the taxpayer’s and the government’s problems, and then comparing the resulting conditions simultaneously, Kolm concludes that “for ex ante public choices, a public pound has a higher social value than a private pound”: (Kolm, p. 269)

\[
U'(W - \theta X) < EU' < V' < U'(W - \theta X - F(W - X)).
\]

The optimal values for the tax rate \( \theta \), the probability of fraud detection \( p \), and the penalty \( F \) are determined by a balance between the utilities of public and private goods.
Srinivasan (1973) relaxed the constant tax rate assumption and introduced the proportionate tax $\theta(W)$ as a function of true income $W$. It is assumed that $\theta(W)>0$, $0<\theta'(W)<1$, and $\theta''(W)\geq 0$; that is, $\theta(W)$ is a positive, increasing, and convex function of $W$. If $\theta''(W)=0$ for all $W$, we get a constant marginal tax rate which together with $\theta(0)=0$ will yield a proportionate rate of tax. If $\theta''(W)>0$ for all $W$, we get a progressive tax structure. The taxpayer chooses the proportion of income, $\lambda$, to be understated. The penalty parameter $F(\lambda)$ is endogenous and is imposed on the evaded income $\lambda W$ as in A-S model.

The individual taxpayer chooses the proportion $\lambda$ so as to maximize his expected income after tax and penalties:

$$E(W)=(1-p)[W-\theta((1-\lambda)W)]+p[W-\theta(W)-F(\lambda)\lambda W].$$

After showing the existence and uniqueness of the optimal proportion $\lambda^*$, Srinivasan derives three main results. The first result states that if the probability of detection $p$ increases, the optimal proportion $\lambda^*$ by which income is understated decreases, i.e. $\partial \lambda^*/\partial p < 0$. The second results says that given a progressive tax function $\theta^* > 0$ and a probability of detection $p$ independent of income, the more income a person generates, the larger the optimal proportion by which he will understate his income, i.e. $\partial \lambda^*/\partial W > 0$. Finally, in the case where $p(W)$ is an increasing function of income, it is hard to say how $\lambda^*$ will respond to changes in $W$ without additional assumptions. Assuming a constant marginal tax rate and $p(W)$, an increasing function of income, the optimal proportion $\lambda^*$ decreases as income increases, i.e. $\partial \lambda^*/\partial W < 0$.

Pencavel (1979) continues to examine the effect of increases in the tax parameters and gross income on the income reporting decisions. He defines a von Neumann-Morgenstern utility function to be a function over total income $Y$ and hours of work $h$, i.e. $U(Y,h)$. Such an assumption illustrates how the tax system can affect not only people’s decision to declare income but also their labor-leisure choice. The individual’s actual tax payments given by $T$ are approximated by a linear continuous function of the form:
\[ T = -S + \theta X^{\sigma}, \]

where \( S \) measures welfare payments from the government to individuals with zero income, \( X \) is the income the individuals report to the tax authorities, \( \theta \) and \( \sigma \) are the parameters capturing the relationship between changes in reported income and changes in tax payments. The existing literature on tax evasion up to date has only analyzed the case of both \( S = 0 \) and \( \sigma = 1 \). The penalty function takes two forms: one is a heavy fine and/or imprisonment plus the payment of the evaded taxes; and the other is some fixed multiplier \( \lambda > 1 \) on the evaded tax payments. The individual makes a simultaneous ex ante decision of how much income to declare to the authorities and how many hours to work. He selects \( X \) and \( h \) so as to maximize the expected utility

\[
EU(Y, h) = (1 - p)U(Y^0, h) + pU(Y^c, h)
\]

subject to

\[
Y^0 = W(h) + S - \theta X^{\sigma}
\]

\[
Y^c = W(h) + S - \theta X^{\sigma} - \left( \frac{\theta(W^{\sigma} - X^{\sigma}) - F}{\lambda \theta(W^{\sigma} - X^{\sigma})} \right)
\]

where \( Y^0 \) is the individual’s net income when he underreports his income but is not caught by the authorities, \( Y^c \) is the taxpayer’s net income when he is caught cheating on his income taxes and then penalized with the fine, and \( p \) is the probability of being audited independent of the decision variables and given the individual’s true taxable income \( W(h) = wh + I \), where \( w \) is the gross hourly wage rate and \( I \) is nonwage income. He considers two scenarios: one is when hours of work is fixed which makes true income be exogenous, and the other is when hours of work are made endogenous.

The first result suggests that when true income is held fixed (\( h \) is fixed) and the risk aversion is a decreasing function of income, an increase in the tax parameters \( \theta \) and \( \sigma \) (in the probability \( p \) of being audited and in the penalty multiplier \( \lambda \)), increases reported income and lowers evasion. Increases in the level of welfare grants \( S \) increases after-tax
wealth and induces a lower level of reported income. Finally, an increase in exogenous true taxable income \( W \) induces a fall in the fraction of income declared.

When true income becomes an endogenous variable (\( h \) is variable), an increase in the tax parameters may induce some ambiguities. For instance, an increase in the penalty rate \( \lambda \) may reduce hours of work which causes a decline in true income which may encourage the taxpayer to report less income to the authorities. An increase in \( \sigma \) and \( \theta \) may also reduce hours of work and thus may reduce true income which may encourage the individual taxpayer to report less income to the tax authorities. In other words, the ambiguities in results derive from induced changes in hours of work and thus true taxable income.

Christiansen (1980) builds a simple theoretical model of tax evasion to analyze whether a large fine (with small probability of detection) is a more effective policy instrument to deter tax evasion than a high probability of detection (with a small penalty). Let \( X \) be the post-tax income if there is no evasion and \( Y \) be the unreported income. The disposable income will then become \( XY + \). If detected, the tax evader will have to pay a fine of \( F \) times the hidden income. The probability of being detected \( p \) is a function of \( F \). Given \( X, p \) and \( F \), the individual chooses the value of \( Y \) to maximize

\[
E = (1 - p)U(X + Y) + pU(X - FY).
\]

Let \( U_s = U(X + Y) \) and \( U_f = U(X - FY) \). The first- and second-order conditions for an interior solution are

\[
E' = pU_s' - (1 - p)U_f' = 0,
\]

\[
E'' = pU_s'' + (1 - p)F^2U_f' < 0.
\]

Using the maximum conditions, Christiansen finds

\[
\frac{dY}{dF} = -\frac{1}{E''} \left[ -(1 - p)U_f' + (1 - p)FYU_f' + (U_f' + FU_f') \frac{dp}{dF} \right].
\]

To analyze the effect of changes in \( F \) and \( p \) on \( Y \), he defines two alternative relationships between \( F \) and \( p \):

\[
1 - p - pF = \text{constant}
\]

and
\( pF = \text{constant.} \)

The first relationship implies that for a given tax evasion the expected gain given by \( pY - (1-p)FY \) is constant and \( dp/dF = (1-p)/(1-F) \). When risk aversion is assumed, \( U'' < 0 \), it implies

\[
\frac{dY}{dF} = \frac{1-p}{E''} \left[ \frac{U'_f}{1+F} - \frac{U'_s}{1+F} - FYU'_f \right] < 0
\]

This result shows that an increase in the penalty rate will discourage tax evasion given that the probability of detection is adjusted to keep the expected gain unchanged. It follows that large fines are more effective to deter tax evasion than a high probability of detection.

Cowell (1985) investigates the phenomenon of “off the books” activities (Ibid p.20). The individual in his model cheats the government by taking one or more different jobs, from which the income is hard to observe and thus tax. Assume that there are three possible choices the person can make: work ‘on the books’, work ‘off the books’, and not to work and rest. Let \( h_0 \) and \( h_1 \) be the person’s proportion of the time he spends in legal and illegal work, respectively. The time left for leisure is \( 1-H \), where \( H = h_0 + h_1 \). The true legal wage rate is \( W_0 \) and the illegal one is \( W_1 \). The government imposes a linear progressive tax system given by

\[
T = \theta y_0 - S,
\]

where \( y_0 \) is taxable income, \( \theta \) is the marginal tax rate, and \( S \) is a lump-sum grant. There are two states of the nature: successful evasion denoted by \( \alpha \), occurred with probability \( (1-p) \), and unsuccessful evasion denoted by \( \beta \), occurred with probability \( p \). The penalty rate is given by \( F \). The person’s disposable income \( c \) in the two states is given by \( c_\alpha \) and \( c_\beta \), respectively, where

\[
c_\alpha = (1-\theta)W_0h_0 + W_1h_1 + S
\]

and

\[
c_\beta = (1-\theta)W_0h_0 + (1-F)W_1h_1 + S.
\]

The individual chooses \( h_0 \) and \( h_1 \) to maximize
\[ V = (1 - p)U(c_\alpha, 1 - H) + pU(c_\beta, 1 - H) \]

subject to \[ c_\alpha = w_\alpha h_0 + W_h h_1 + S \]
\[ c_\beta = w_\beta h_0 + w_i h_1 + S, \]

where \( w_0 = (1 - \theta)W_0 \) and \( w_i = (1 - F)W_i \).

Unlike previously discussed models on tax evasion, Cowell makes agents not choose how much income to declare but how much time to allocate between legal and illegal activities and leisure. He claims that the fact that labor decisions are endogenous produces conflicting results and leads to no straightforward comparative statics conclusion. As long as \( U \) is additively separable in consumption and leisure and labor supply is backward-bending he could draw some conclusion about the level of evasion activity \( h_1 \). He concludes that if labor supply is backward bending, if absolute risk aversion is decreasing and relative risk aversion is increasing, then evasion activity increases with increases in \( \theta \).

Rausch (1991) looks at the relationship between the size of the informal sector and the minimum wage. His general equilibrium model, based on Lucas (1978) “span of control framework”, suggests that entrepreneurs go underground to avoid minimum wage, which is greater than the paying market wage.

Dunn (1992) claims that the penalty for tax evasion would achieve only a modest change in tax compliance. Using Mexican data for 1982-89, his estimated results show that “a doubling of the fine for tax evasion would increase declared taxable income by about 10 percent” (Ibid p.14).

Loayaza (1996) claims there are two types of regulations that influence the size of the informal labor market: red tape and taxes. Red tape and bureaucratic extortion (bribing) make starting a new business officially an unattractive option and can lead new firms to the informal sector. On the other hand, taxes and redundancy pay make official firms offer lower wages which drives potential employees away into underground jobs. Depending on various combinations of policy parameters, equilibria with different share of the informal economy are possible. The paper draws conclusions regarding the role of unemployment benefits in reducing the size of the informal economy. Policy implications are offered.
Djankov et al. (2002) analyze the consequences of the regulation of entry in 85 countries. They describe legal procedures to start up a business, as well as the official time and the official cost of meeting these procedures. Their theory of regulatory entry predicts that stricter regulation of entry is associated with sharply higher corruption and larger unofficial economies, but not better quality products. Countries with more democratic and limited governments have lighter regulation of entry.

Antunes and Cavalcanti (2006) build a model in which the agents choose the sector – formal or informal – to operate in. Those who choose to operate formally face the trade-off between the entry costs and the benefit of a better access to credit institutions. They apply their model to explain the observed differences in the size of informal sector between the United States and a Southern European country³ and the Peruvian economy⁴.

They conclude that a way to reduce the informal sector size in the Southern European economy would be to lower the regulation costs such as taxes, legal fees, bribes and other implicit costs. As they lower the regulations costs the entrepreneurs bear, more entrepreneurs find it profitable to switch to the formal sector. The access to credit institutions enables them to expand their business even further. This, in turn, increases labor demand and the wage rate. Higher wages encourages some entrepreneurs to leave their current positions and become workers. As a result, the informal sector shrinks. However, for developing countries, both the barriers to entry and the level of contract enforcement have comparable importance in shifting firm activity from the informal to the formal sector.

The models in this section are all static by nature. They attempt to answer the question of how much income the individuals decide to reveal to the authorities in the face of the known penalties and the known possibility of being detected. They analyze the effects of tax parameters, the structure of penalties and the probability of being detected on the amount of tax evasion. The results state that an increase in the tax rate or the probability of getting caught will increase the proportion of income declared and reduce tax evasion.

³ The Southern Europe economy is a synthesis of the economies of Italy, Portugal and Spain.
⁴ The Peruvian economy epitomizes Latin-American economies.
3 Data

From late 1998 to mid-2000 the World Bank sponsored personal interviews with managers of more than 10,000 enterprises in 80 countries covering the main regions of the world – The World Business Environment Surveys (WBES). The WBES uses a uniform core questionnaire to registered businesses to capture companies’ perceptions of key constraints in the business environment posed by taxation, government regulations, corruption of public officials, functioning of the judiciary, and access to financial surveys. A more detailed description of the survey can be found in Batra et al. (2003).

Of interest to this study, the survey included a question regarding the extent and intensity to which firms fail to report income to the tax authority, which permits studies exploring the links between tax non-compliance and various firm characteristics. Specifically, the managers were asked, “Recognizing the difficulties that many firms face in fully complying with taxes and regulations, what percent of total annual sales would you estimate the typical firm in your area of business reports to tax authorities.” There were seven possible answers: 1 = all (100%), 2 = 90-99%, 3 = 80-89%, 4 = 70-79%, 5 = 60-69%, 6 = 50-59%, and 7 = less than 50%. The distribution of answers to the sales reporting question over the entire sample is given in Table 1.

As Table 1 shows, underreporting was clearly perceived to be greater among small firms than either medium-size or large firms. Only 25.8 percent of small firms said that firms like them report 100 percent of sales (or income). The conclusion is that around the world, informality is negatively associated with firms’ productivity and their size.

4 The Model

In this section I introduce a static model of tax evasion. This model incorporates the assumptions of product differentiation and firm productivity heterogeneity using the monopolistic competition framework proposed by Melitz (2003).

Consider an economy with a continuum of mass one of identical consumers-workers, each of whom is endowed with \( \bar{l} \) units of labor. There is a measure of \( n \) potential firms. The representative consumer derives utility from the consumption of the differentiated
goods according to a symmetric CES utility function. There is a continuum of the differentiated goods. Each variety is produced by a single firm. Firms differ ex-ante only in their labor productivities indexed by $x \in X$ (which is also used as the index for varieties). Firm productivities are distributed on the interval $x \geq 1$ according to the Pareto distribution with distribution function

$$F(x) = 1 - x^{-\gamma},$$

which has the density function

$$f(x) = \gamma x^{-(\gamma+1)}.$$  

In other words, I suppose that the probability density function (p.d.f) of a random variable $X$ with a continuous distribution is

$$f(x) = \begin{cases} \gamma x^{-(\gamma+1)} & \text{for } x \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

where shape parameter $\gamma > \sigma - 1$.

Firms in the economy face two decisions: produce or shut down, pay or evade taxes. The timing of the firm’s decisions is as follows. An entering firm first invests a fixed entry-cost $f$ to receive efficiency $x$. (Notice that firms have potentially different productivity levels $x$.) After $x$ is revealed, a firm may decide to shut down and incur no further costs. If the firm decides to produce then it may choose to continue with future production and either pay taxes or evade taxes. Once the decision is made, it cannot be altered. Legal firms are subject to a payroll tax, $\tau$. For each dollar of gross income, then, an honest firm would receive $(1 - \tau)$ dollars. If the firm was dishonest, did not pay tax liabilities and escaped detection, it would receive 1 dollar. But if it got caught, it would face a fixed penalty $\zeta$ associated with tax regulations.

### 4.1 Consumers

Let $\bar{x}$ be the minimal cutoff productivity level such that any entering firm drawing a productivity level $x < \bar{x}$ will immediately exit and never produce. Once the firm enters the market with efficiency level $x \geq \bar{x}$, it produces a final good $c(x)$ to satisfy
consumers’ demand. The representative consumer derives utility from consumption according to the logarithmic CES Dixit-Stiglitz utility function of the form:

\[ U = \log \left( n \int_{\bar{x}}^{\infty} c(x)^\rho \, dF(x) \right)^{1/\rho}, \]  

(4)

where the parameter \( \rho, 0 < \rho \leq 1 \), governs the elasticity of substitution \( 1/(1-\rho) \) between any two differentiated consumption goods. The consumer faces the following budget constraint:

\[ n \int_{\bar{x}}^{\infty} p(x)c(x) \, dF(x) = w\bar{l} + \Pi + R, \]  

(5)

where \( c(x) \) is consumption goods produced by firms with productivities levels \( \bar{x} \leq x < \infty \) and \( p(x) \) is the price charged by the firms. The consumer supplies \( \bar{l} \) units of labor at the nominal wage rate, \( w \). In addition to labor income, the consumer receives the profit flows \( \Pi \) of firms and the lump-sum transfers \( R \). Both the aggregate profits \( \Pi \) and lump-sum transfers \( R \) are rebated equally to all consumers. The consumers’ ownership of the firms is modeled as passive, in that they take the profit rebate as given. Similarly, the lump-sum transfers \( R \) are given too.

The consumer’s problem is to choose \( c(x) \) to maximize (4) subject to (5). The first order conditions of this problem include

\[ c(x') = \left( \frac{p(x')}{p(x)} \right)^{1/(\rho-1)} c(x), \]  

(6)

\[ n \int_{\bar{x}}^{\infty} p(x)c(x) \, dF(x) = w\bar{l} + \Pi + R. \]  

(7)

The solution to the maximization problem of the consumer gives rise to the usual CES Dixit-Stiglitz demands for each \( x \) according to:

\[ c(x) = \frac{w\bar{l} + \Pi + R}{p(x)^{1-\rho} \left( p(x)^{1-\rho} \right)} \]  

(8)
where \( \Pi, R, \) and \( P \) are the aggregate profit, aggregate transfer flows, and overall price level, respectively.

In an equilibrium characterized by a mass \( n \) of firms (and hence \( n \) goods) and a distribution \( f(x) \) of productivity level over a subset of \( (1, \infty) \), the aggregate price level \( P \) can be found by solving a cost minimization problem of acquiring one unit of aggregate consumption good given by:

\[
\min \left[ n \int_{x}^{\infty} p(x) c(x) dF(x) \right]
\]

subject to

\[
\left( n \int_{x}^{\infty} c(x)^{\rho} dF(x) \right)^{1/\rho} = 1
\]

The aggregating price level derived from (9) and (10) is given as follows:

\[
P^{\frac{\rho}{1-\rho}} = \left( n \int_{x}^{\infty} p(x)^{\frac{\rho}{\rho-1}} dF(x) \right)
\]

### 4.2 Firms

In this section, I specify the maximization problems of the firms in the economy and characterize their solutions. Production involves a fixed and variable cost each period. At the beginning of a period, each firm chooses its labor input. Labor is the only factor of production, and I normalize the wage level to unity. Firms produce with a technology exhibiting constant marginal cost \( 1/x \), along with an overhead per-period fixed cost \( f \) (measured in labor units). Given the direct demand and a continuum of competing firms, all firms set a constant markup \( \sigma/(\sigma-1) \) over marginal cost. The technology used by a firm with efficiency \( x \) combines labor inputs \( l \) to produce output \( y \) according to:

\[
y(x) = \max \left[ x(l(x) - f), 0 \right].
\]

Here \( f > 0 \) is the level of fixed costs. Production then occurs, and each firm chooses whether to pay taxes or evade them.
4.2.1 Taxpayer Firms

The profit maximization problem of a taxpayer firm with efficiency \( x \) is:

\[
\pi^T(y^T(x)) = \max \left[ p^T(x)y^T(x) - (1 + \tau)wT^T(x) \right]
\]

subject to

\[
y^T(x) = \max \left[ x\left( I^T(x) - f \right), 0 \right]
\]

Here \( I^T(x) \) and \( y^T(x) \) denote the amount of labor and output of the taxpayer firm, respectively. The payroll tax rate is indexed by uniform payroll tax rate, \( \tau \). It is clear that an operating firm would choose \( y^T(x) = x\left( I^T(x) - f \right) \). Using the market clearing condition, we know that \( y^T(x) = c^T(x) \). Given the direct demand function \( c^T(x) \) that comes from the consumer’s problem, the profit maximization problem given by (13) and (14) can be re-written to reflect this:

\[
\max_{p(x)} p^T(x)c^T(x) - (1 + \tau)w\left( \frac{c^T(x)}{x} + f \right).
\]

Assume that the wage rate \( w \) is equal to one. This yields an optimal pricing rule for \( c^T(x) \) given as follows:

\[
p^T(x) = \frac{1 + \tau}{\rho x}.
\]

Equation (16) illustrates the fact that among taxpayers, a more productive firm (higher \( x \)) will be bigger, charge a lower price, and earn higher profits than a less productive firm.

4.2.2 Non-taxpayer Firms

Here, I describe the profit maximization problem for a non-taxpayer firm with efficiency \( x \). I assume that the probability of being monitored depends monotonically on the size of the firm. There are several possibilities for measuring the size of the firm – output, labor force or productivity level. Here I choose to use the firm’s output. Let the probability of being detected \( 1 - \mu(y(x)) \) be defined as follows:
\[
1 - \mu(y(x)) = \begin{cases} 
0 & \text{if } y(x) \leq \bar{y}_L \\
1 - \bar{\mu} & \text{if } \bar{y}_L < y(x) \leq \bar{y}_H \\
1 & \text{if } y(x) > \bar{y}_H
\end{cases}
\]

(17)

where \(0 < \bar{\mu} < 1\), \(\bar{y}_L\) and \(\bar{y}_H\) are the low and high exogenously given levels of output that the government sets for its monitoring purposes. The functional form is chosen so that the firms producing less than or equal to \(\bar{y}_L\) decide to evade taxes since they know they won’t be monitored. It is costly for the government to monitor and prosecute small firms. The firms with productivity levels \(y(x)\) such that \(y(x) > \bar{y}_L\) and \(y(x) \leq \bar{y}_H\) produce final output \(y(x)\) and face a constant chance of being caught, expressed as \(1 - \bar{\mu}\). Lastly, when the firm’s production exceeds \(\bar{y}_H\), the evading firm will certainly be caught by the authorities for tax evasion, thus such firms pay all taxes.

The non-taxpayer sets its price \(p_{NT}^x\) to solve the profit maximization problem given by:

\[
\max_{p_{NT}^x} p_{NT}^x y_{NT}^x - w \left[ \frac{y_{NT}^x}{x} + f \right] - \left(1 - \mu(y(x)) \right) \left[ \tau w \left( \frac{y_{NT}^x}{x} + f \right) + \zeta \right]
\]

(18)

subject to

\[
y_{NT}^x = \max \left[ x (l_{NT}^x - f), 0 \right]
\]

(19)

If caught, the non-taxpayer will face a fixed cost \(\zeta\) associated with operating illegally, as well as payments of taxes owed. Analogously, the direct demand function \(c_{NT}^x\) that comes from the consumer’s problem is equal to \(y_{NT}^x\) by the market clearing condition. Using (8) and solving (18) and (19), I find the optimal pricing rule for the non-taxpayer given by:

\[
p_{NT}^x = \frac{1 + \left(1 - \mu(y_{NT}^x) \right) \tau}{\rho x}
\]

(20)

As in (16), the pricing rule \(p_{NT}^x\) is inversely related to the efficiency level. The only difference is that tax evaders that are not monitored by government officials are no longer
subject to taxes. It allows them to charge lower price $\frac{1}{\rho x}$ and earn higher profits than taxpayers. Once the firms start to be monitored with probability $1-\bar{\mu}$, Equation (20) becomes

$$p_{\mu}^{NT}(x) = \frac{1+(1-\bar{\mu})\tau}{\rho x}.$$

(21)

4.3 Efficiency and Decision-Making

This section determines the crucial elements of the model such as the cutoff productivity levels for entry-exit and tax evasion decisions. I will discuss each one separately.

4.3.1 Efficiency and Entry-Exit Decision

Each industry has multiple potential producers of each good with varying level of efficiency $x$. Except for this heterogeneity in efficiency, the production technology is identical across producers regardless of where and what they produce. Prior to entry, firms are identical. To enter, firms must first pay a fixed entry cost $f > 0$ in units of labor costs. Firms then draw their initial productivity parameter $x$ from a Pareto distribution $f(x)$. If the firm draws a low productivity draw, it immediately exits. Assume that $\bar{x}_{NT}$ is the productivity cutoff level for operating firms. If the firm produces, its productivity level is $x \geq \bar{x}_{NT}$. As in Melitz (2003), this simplification highlights the fact that new entrants will experience, on average, lower productivity and a higher probability of exit than incumbents.

Any entering firm with productivity $x$ would immediately exit if it earns a negative profit $\pi(x) < 0$. No firm would want to enter if the value were negative. All other incumbent firms would produce and earn $\pi(x) \geq 0$. Assume that the cutoff firm drawing the smallest productivity level $\bar{x}_{NT}$ is not monitored by the tax administration
(meaning $\mu \left( y^{NT} \left( x_1^{-NT} \right) \right) = 1$) and earns a zero profit. Then the productivity cutoff level $x_1^{-NT}$ is then determined by the zero cutoff profit condition

$$\pi^{NT} \left( y^{NT} \left( x_1^{-NT} \right) \right) = 0.$$  \hspace{1cm} (22)

It implies that

$$\frac{f_n x \Phi}{(1 + \tau)^{\frac{\rho}{1 - \rho}} \rho (wl + \Pi + R)(\gamma(1 - \rho) - \rho)} \left[ \frac{1 - \rho}{\rho} \right]^{-1}, \hspace{1cm} (23)$$

where

$$\Phi = (1 + \tau)^{\frac{\rho}{1 - \rho}} \left( \left( x_1^{-NT} \right)^{\nu} - \left( x_2^{-NT} \right)^{\nu} \right) + \gamma(1 - \rho) - \rho \left( (1 + \tau)^{\frac{\rho}{1 - \rho}} \left( x_2^{-NT} \right)^{\nu} - \left( x_1^{-NT} \right)^{\nu} \right) +$$

$$\frac{1 + \tau}{\psi} \left( x_1^{-NT} \right)^{\nu} - \left( x_2^{-NT} \right)^{\nu} + \frac{\gamma(1 - \rho) - \rho}{\gamma(1 - \rho)} \left( \frac{1 + \tau}{\psi} \right)^{\frac{\rho}{1 - \rho}} \left( x_2^{-NT} \right)^{\nu} - \left( x_1^{-NT} \right)^{\nu} +$$

and $\psi = 1 + (1 - \mu)\tau$ and $\nu = \rho/(1 - \rho) - \gamma$.

Equation (23) says that any firm drawing a productivity level $x < x_1^{-NT}$ will immediately exit and never produce. Any firm with $x \geq x_1^{-NT}$ will enter and produce. We can also see that the first cutoff threshold is negatively correlated with taxes: the higher the tax rate, the lower the threshold will become, and more small firms decide to enter and go underground. However, the higher the entry costs $f$, the lower the threshold will become, causing fewer firms to enter the market.

### 4.3.2 Efficiency and Tax Evasion Decision

Any operating firm always seeks to maximize its profits. It follows the same rule even when deciding whether to pay or evade taxes. In this paper I assume that greater production outcome raises the probability of detection. If a firm’s production level $y(x)$
has exceeded a pre-determined level of $\bar{y}_L$, there is a $1 - \bar{\mu}$ - percent chance of being monitored by the government. If so, the firms with productivity levels $x$ such as $x \geq x_1^{NT}$ and $x \leq x_2^{NT}$ produce $y^{NT}(x)$ given by $\frac{w\hat{I} + \Pi + R}{p^{NT}(x)}$ and face no risk of being caught for tax evasion. The model become less trivial when firms reach the first critical threshold $\bar{x}_2^{NT}$ that corresponds to the production level $\bar{y}_L$. The firms with productivity levels $x$ such as $x > \bar{x}_2^{NT}$ and $x \leq \bar{x}_1^{NT}$ can potentially produce more than $y_L$ and face a $1 - \mu$ - percent chance to be caught for tax evasion. In this paper I will model the case when such firms decide to scale down their production and fix it exactly at $\bar{y}_L$ as if they were producers of type $\bar{x}_2^{NT}$ and charge a price $p^{NT}(\bar{x}_2^{NT})$. By doing so, they avoid being penalized for tax evasion. Since the firms’ efficiency levels keep increasing on this interval, each firm is able to produce $\bar{y}_L$ with a fewer number of workers. A reduction in firms’ employment level enables them to lower their payroll tax payments and a fixed price increase increases their operating revenues. The cutoff level $\bar{x}_2^{NT}$ is determined by the following condition:

$$\pi^{NT}(y^{NT}(\bar{x}_2^{NT})) = \pi^{NT}(\bar{y}_L),$$  \hspace{1cm} (24)

which is the same as $y^{NT}(\bar{x}_2^{NT}) = \bar{y}_L$.

Then the optimal cutoff rule implies

$$\bar{x}_2^{NT} = \left( \frac{\bar{y}_L n(1 - \rho)\zeta\Phi}{(1 + \tau)^{-\rho}(wI + \Pi + R)(\gamma(1 - \rho) - \rho)\rho} \right)^{1 - \rho} \hspace{1cm} (25)$$

Equation (25) shows that the second cutoff level $\bar{x}_2^{NT}$ is also negatively correlated with the tax rate having all other factors fixed. But at the same time we can see that it is positively correlated with the lower bound level $\bar{y}_L$. So as $\bar{y}_L$ increases, so does $\bar{x}_2^{NT}$, and a higher percentage of small firms decide to go underground.
The third cutoff level $\overline{x}_1$ is determined in a similar fashion but now the firm must be indifferent in profits between producing a fixed level $\overline{y}_L$ with a zero probability of being monitored and a level $y^{NT}_{\overline{x}}\left(\overline{x}_1\right)$ with a $1-\bar{\mu}$ percent chance to be caught. In other words, the threshold $\overline{x}_1$ is determined by the following condition:

$$\pi^{NT}\left(\overline{y}_L\right) = \pi^{NT}_{\overline{x}}\left(y^{NT}_{\overline{x}}\left(\overline{x}_1\right)\right),$$

(26)

where $y^{NT}_{\overline{x}}\left(\overline{x}_1\right) = \frac{1 + \tau}{\psi^{\overline{x}}\left(1 - \rho\right)\Phi} \frac{\rho}{\psi^{\overline{x}}\left(1 - \rho\right)\Phi} \left(1 + \tau\right)^{1 - \rho} \left(w + \Pi + R\right) \left(y^{NT}_{\overline{x}}\left(\overline{x}_1\right)\right)^{1 - \rho}.$

Equation (26) can be re-written as follows:

$$p^{NT}\left(\overline{y}_L\right) = \frac{y^{NT}_{\overline{x}}\left(\overline{x}_1\right) + f}{\frac{y^{NT}_{\overline{x}}\left(\overline{x}_1\right) + f}{1 - \bar{\mu}} \left(1 + \tau\right)^{1 - \rho} \left(w + \Pi + R\right) \left(y^{NT}_{\overline{x}}\left(\overline{x}_1\right)\right)^{1 - \rho} + f + \zeta}.$$  

(27)

Equation (27) implies

$$\overline{x}_1 = \frac{1 - \rho}{\rho} y^{NT}_{\overline{x}}\left(\overline{x}_1\right) + \frac{\overline{y}_L}{\rho \overline{x}_2} + \left(1 - \bar{\mu}\right)[\tau f + \zeta].$$

(28)

The fourth cutoff level $\overline{x}_2$ is such that the firm must be indifferent in profits between producing $y^{NT}_{\overline{x}}\left(\overline{x}_1\right)$ and producing a higher threshold level $\overline{y}_H$ with a $1 - \bar{\mu}$ probability of being caught in both cases. In other words, the threshold $\overline{x}_2$ is determined by the following condition:

$$\pi^{NT}_{\overline{x}}\left(y^{NT}_{\overline{x}}\left(\overline{x}_2\right)\right) = \pi^{NT}_{\overline{x}}\left(\overline{y}_H\right),$$

(29)

which is equivalent to $y^{NT}_{\overline{x}}\left(\overline{x}_2\right) = \overline{y}_H.$
Equation (29) implies
\[
\frac{1}{x_2} = \frac{y_{\mu} \psi^{1-\rho} n(1-\rho) \gamma \Phi}{(1+\tau)^{1-\rho}(wl + \Pi + R)(\gamma(1-\rho)-\rho)}^{1-\rho}.
\] (30)

Finally, the firms keep producing at \( y_{\mu} \) until the point they are indifferent between not paying all taxes and being caught with a probability \( 1-\mu \) and paying all taxes and producing at \( y^T \). That can be written as
\[
\pi^{NT}_{\mu}(y_{\mu}) = \pi^T(y^T(x)),
\] (31)
where \( y^T(x) = \frac{(wl + \Pi + R)(\gamma(1-\rho)-\rho)\rho}{n(1-\rho)\gamma(1+\tau)\Phi} \).

Equation (31) becomes
\[
p^{NT}_{\mu}(y_{\mu}) = \left(\frac{y_{\mu}}{x} + f \right) - (1-\mu) \left( \tau \left( \frac{y_{\mu}}{x} + f \right) + \zeta \right) =
\] (32)
\[p^{T}(x)c^{T}(x) - (1+\tau) \left( \frac{c^{T}(x)}{x} + f \right)\]

Thus, the threshold \( x \) is defined as
\[
\frac{n\gamma\Phi \left[ \psi^{1-\rho} n(1-\rho) \gamma \Phi \left( y_{\mu} - \frac{1}{x_2} \right) + (1+\tau-\psi)f - (1-\mu) \right]}{(wl + \Pi + R)(\gamma(1-\rho)-\rho)}^{1-\rho}.
\] (33)

4.4 Aggregation

In equilibrium, the aggregate profit \( \Pi \) is given by
\[
\Pi = \Pi_1 + \Pi_2 + \Pi_3 + \Pi_4 + \Pi_5,
\] (34)
where
\[ \Pi_1 = n \int_{\tilde{x}^N} \pi^{NT} \left( y^{NT} (x) \right) dF(x) = \]

\[
\frac{(1 + \tau)\frac{\rho}{\Phi} \left( \tilde{\tau} + \Pi + R \right) \left( 1 - \rho \right)}{\Phi} \left[ \left( \tilde{x}_1 \right)^{\gamma} - \left( \tilde{x}_2 \right)^{\gamma} \right] - n \left[ \tilde{x}_1 \right] - \left( \tilde{x}_2 \right)^{\gamma} ;
\]

\[ \Pi_2 = n \int_{\tilde{x}^F} \pi^{NT} \left( \tilde{y}_L \right) dF(x) = \]

\[
n \left[ \frac{(1 - \rho)}{\Phi} \left( \tilde{\tau} \right) \left( \tilde{\tau} + \Pi + R \right) \left( 1 - \rho \right) \left[ \left( \tilde{x}_1 \right)^{\gamma} - \left( \tilde{x}_2 \right)^{\gamma} \right] - n \left[ \tilde{x}_1 \right] - \left( \tilde{x}_2 \right)^{\gamma} ;
\]

\[ \Pi_3 = n \int_{\tilde{x}^N} \pi^{NT}_p \left( y^{NT}_H (x) \right) dF(x) = \]

\[
\frac{(1 + \tau)\frac{\rho}{\Phi} \left( \tilde{\tau} + \Pi + R \right) \left( 1 - \rho \right)}{\Phi} \left[ \left( \tilde{x}_1 \right)^{\gamma} - \left( \tilde{x}_2 \right)^{\gamma} \right] - n \left[ \tilde{y}_H \right] - (1 - \tilde{\mu}) \left[ \tilde{y}_H \right] - \left( \tilde{x}_2 \right)^{\gamma} ;
\]

\[ \Pi_4 = n \int_{\tilde{x}^H} \pi^{NT}_H \left( y^H (x) \right) dF(x) = \]

\[
n \left[ \frac{(1 - \rho)}{\Phi} \left( \tilde{\tau} \right) \left( \tilde{\tau} + \Pi + R \right) \left( 1 - \rho \right) \left[ \left( \tilde{x}_1 \right)^{\gamma} - \left( \tilde{x}_2 \right)^{\gamma} \right] - n \left[ \tilde{y}^H \right] - (1 - \tilde{\mu}) \left[ \tilde{y}^H \right] - \left( \tilde{x}_2 \right)^{\gamma} ;
\]

\[ \Pi_5 = n \int_{\tilde{x}^T} \pi^T \left( y^T (x) \right) dF(x) = \]

\[
\frac{(1 + \tau)\frac{\rho}{\Phi} \left( \tilde{\tau} + \Pi + R \right) \left( 1 - \rho \right) \left[ \left( \tilde{x}_1 \right)^{\gamma} - \left( \tilde{x}_2 \right)^{\gamma} \right] - n (1 + \tau) \left[ \tilde{x}^\gamma \right] ,
\]

where \( \pi^{NT} \left( y^{NT} (x) \right) \), \( \pi^{NT} \left( \tilde{y}_L \right) \), \( \pi^{NT}_p \left( y^{NT}_H (x) \right) \), \( \pi^{NT}_p \left( \tilde{y}_H \right) \), and \( \pi^T \left( y^T (x) \right) \) are aggregate profits for non-taxpayers producing \( y^{NT} (x) \), non-taxpayers producing \( \tilde{y}_L \), non-taxpayers producing \( y^{NT}_H (x) \) and being monitored with probability \( 1 - \tilde{\mu} \), non-taxpayers producing \( \tilde{y}_H \) and also being monitored with probability \( 1 - \tilde{\mu} \), and taxpayers producing \( y^T (x) \), respectively. The first and second integrals measure the average profits of the tax...
evading firms which are not monitored and are the ones in the range of \( x \) starting from \( \bar{x}_1 \) to \( \bar{x}_2 \) and from \( \bar{x}_2 \) to \( \bar{x}_1 \), respectively. The third and fourth integrals measure the average profits of the tax evading firms which are monitored with probability \( 1 - \bar{\mu} \) and have efficiency levels \( x \) starting from \( \bar{x}_1 \) to \( \bar{x}_2 \) and from \( \bar{x}_2 \) to \( \bar{x} \), respectively. Finally, the fourth integral measures the average profit of all firms that pay taxes with \( x > \bar{x} \).

In equilibrium, the aggregate transfers \( R \) is given by

\[
R = R_1 + R_2 + R_3 + R_4 + R_5,
\]

where

\[
R_1 = n \int_{\bar{x}_1}^{\bar{x}_2} r^{NT} \left( y^{NT} \left( x \right) \right) dF \left( x \right), \quad R_2 = n \int_{\bar{x}_2}^{\bar{x}_1} r^{T} \left( y_L \right) dF \left( x \right), \quad R_3 = n \int_{\bar{x}_1}^{\bar{x}_2} r\left( y_T^{NT} \left( x \right) \right) dF \left( x \right); \quad R_4 = n \int_{\bar{x}_2}^{\bar{x}} r^{NT} \left( y_H \right) dF \left( x \right), \quad R_5 = n \int_{\bar{x}}^{\infty} r^{T} \left( y \right) dF \left( x \right).
\]

Notice that the expression inside the first integral given by

\[
\int_{\bar{x}_1}^{\bar{x}_2} r^{NT} \left( y^{NT} \left( x \right) \right) dF \left( x \right) = \left( 1 - \mu \left( y \left( x \right) \right) \right) \left[ \tau w \left( y^{NT} \left( x \right) / x + f \right) + \zeta \right]
\]

stands for the amount of taxes the individual tax evader producing \( y^{NT} \left( x \right) \) pays if caught. Similarly, the expression inside the second integral

\[
\int_{\bar{x}_2}^{\bar{x}_1} r^{T} \left( y_L \right) dF \left( x \right) = \left( 1 - \mu \left( y_L \right) \right) \left[ \tau w \left( y_L / x + f \right) + \zeta \right]
\]

is the amount of taxes the firm with \( y_L \) pays if caught. The probability of tax detection in both cases is zero which leads to \( R_1 = R_2 = 0 \) as a result. The firms which produce more than \( y_L \) start to be monitored. Thus, \( R_3 \) becomes as follows

\[
R_3 = \left( 1 - \bar{\mu} \right) \tau \left( 1 + \bar{\tau} \right)^{\beta} \left( w + \bar{\Pi} + \bar{R} \right)^{\beta} \Phi \left[ \left( \bar{x}_1 \right)^{\gamma} - \left( \bar{x}_2 \right)^{\gamma} \right] + n \left( 1 - \bar{\mu} \right) \left[ \tau f + \zeta \right] \left[ \left( \bar{x}_1 \right)^{\gamma} - \left( \bar{x}_2 \right)^{\gamma} \right].
\]

The transfers \( R_4 \), which are nothing but the tax revenue received from the firms producing at a level of \( y_H \), are given by:

\[
R_4 = n \left( 1 - \bar{\mu} \right) \left[ \tau y_H \frac{\gamma}{\gamma + 1} \left[ \left( \bar{x}_1 \right)^{-(\gamma+1)} - \left( \bar{x}_2 \right)^{-(\gamma+1)} \right] + \left[ \left( \bar{x}_2 \right)^{-\gamma} - \left( \bar{x} \right)^{-\gamma} \right] (\tau f + \zeta) \right].
\]
Lastly, the expression $r^T(x) = \tau w\left(y^T(x)/(x + f)\right)$ is the regular tax payments paid by the honest firm. This will lead to

$$R_s = \frac{\tau(wl + \Pi + R)\rho}{(1 + \tau)\Phi} \left\{\frac{1}{\rho} + n\tau f\left(\frac{1}{x}\right)^\gamma\right\}.$$

In equilibrium, the aggregate price level $P$ is given by

$$P = \left(P_1 + P_2 + P_3 + P_4 + P_5\right)\frac{\rho - 1}{\rho}, \quad (36)$$

where

$$P_1 = n \int_{\frac{x_1}{x_1^T}} \int p^{NT}(x)\frac{\rho}{\rho - 1} dF(x) = n \int_{\frac{x_1}{x_1^T}} \int \frac{1}{\rho x} \gamma x^{-\gamma - 1} dx;$$

$$P_2 = n \int_{\frac{x_1}{x_1^T}} \int p^{NT}\left(\frac{x_2}{x_2^T}\right)\frac{\rho}{\rho - 1} dF(x) = n \int_{\frac{x_1}{x_1^T}} \int \frac{1}{\rho x} \gamma x^{-\gamma - 1} dx;$$

$$P_3 = n \int_{\frac{x_1}{x_1^T}} \int p^{NT}(x)\frac{\rho}{\rho - 1} dF(x) = n \int_{\frac{x_1}{x_1^T}} \int \frac{1}{\rho x} \gamma x^{-\gamma - 1} dx;$$

$$P_4 = n \int_{\frac{x_1}{x_1^T}} \int p^{NT}(x)\frac{\rho}{\rho - 1} dF(x) = n \int_{\frac{x_1}{x_1^T}} \int \frac{1}{\rho x} \gamma x^{-\gamma - 1} dx;$$

$$P_5 = n \int_{\frac{x_1}{x_1^T}} \int p^{NT}(x)\frac{\rho}{\rho - 1} dF(x) = n \int_{\frac{x_1}{x_1^T}} \int \frac{1}{\rho x} \gamma x^{-\gamma - 1} dx.$$
\[ c^{NT}(x) = \frac{(1 + \tau)^{\frac{\rho}{1 - \rho}}(w\hat{\lambda} + \Pi + R)(\gamma(1 - \rho) - \rho)x^{-\frac{1}{1 - \rho}}}{n(1 - \rho)\Phi} \]  \hspace{1cm} (39) 

and

\[ c_{\mu}^{NT}(x) = \frac{(1 + \tau)^{\frac{\mu}{1 - \mu}}(w\hat{\lambda} + \Pi + R)(\gamma(1 - \rho) - \rho)x^{-\frac{1}{1 - \rho}}}{(1 + (1 - \mu) \tau)^{-\frac{1}{1 - \mu}} n(1 - \rho)\Phi}. \]  \hspace{1cm} (40)

### 4.5 Definition of Equilibrium

The equilibrium for this economy is specified by a set of all the endogenous variables included in the model economy. That is, price functions \( p^{NT}(x), p^{NT}_\mu(x), \) and \( p^T(x) \) for each productivity level \( x \); a nominal wage rate \( \hat{w} \); a consumption plan for consumers \( c^N(x), c^N_\mu(x), \) and \( c^T(x) \); a production plan for producers \( y^N(x), l^N(x) \), \( y^N_\mu(x), l^N_\mu(x) \), and \( l^T(x) \); aggregate profit flows \( \hat{\Pi} \); aggregate transfers \( \hat{R} \); a cutoff productivity level \( \overline{x}^{NT}_1 \) such that firms drawing a productivity level \( x \) such as \( x \geq \overline{x}^{NT}_1 \) enter the market and with \( x < \overline{x}^{NT}_1 \) immediately exit the market; a cutoff tax evasion level \( \overline{x}^{NT}_2 \) such that firms with \( \overline{x}^{NT}_1 \leq x \leq \overline{x}^{NT}_2 \) produce \( y^{NT}(x) \), evade taxes and are not caught; a cutoff tax evasion level \( \overline{x}^{T}_1 \) such that firms with \( \overline{x}^{NT}_2 < x \leq \overline{x}^{T}_1 \) evade taxes and produce a fixed level of \( \overline{y}_L \); a cutoff tax evasion level \( \overline{x}^{T}_2 \) such that firms with \( \overline{x}^{T}_1 < x \leq \overline{x}^{T}_2 \) produce \( y^{NT}_\mu(x) \), evade taxes and are at risk to be caught with probability \( 1 - \overline{\mu} \); a cutoff tax evasion level \( \overline{x} \) such that firms with \( \overline{x}_2 < x \leq \overline{x} \) evade taxes and produce a fixed level of \( \overline{y}_\mu \) and are also at risk to be caught for tax evasion with probability \( 1 - \overline{\mu} \); lastly, firms with a productivity level \( x > \overline{x} \) produce \( y^T(x) \) and pay taxes such that:
1. Given $p^{NT}(x)$, $p^{NT}_{\mu}(x)$, $p^{T}(x)$, $w$, $\hat{\Pi}$, $\hat{R}$, the individual consumption plans $c^{NT}(x)$, $c^{NT}_{\mu}(x)$, and $c^{T}(x)$ solve the utility maximization problem of the representative consumer, that is, maximize (4) subject to (5);

2. Given the direct demand function $c^{T}(x)$ that comes from the consumer’s problem, the taxpayer firm chooses $p^{T}(x)$ to solve (15);

3. Given the direct demand function $c^{NT}(x)$ that comes from the consumer’s problem, the non-taxpayer firm chooses $p^{NT}(x)$ to maximize (18) subject to (19);

4. Given the direct demand function $c^{NT}_{\mu}(x)$ that comes from the consumer’s problem, the non-taxpayer firm chooses $p^{NT}_{\mu}(x)$ to maximize (18) subject to (19);

5. The markets for goods clear:

$$\hat{y}^{i}(x) = c^{i}(x), \; i = NT, T; \tag{41}$$

$$\hat{y}^{NT}(x) = c^{NT}(x) \tag{42};$$

$$\bar{y}_{L} = c_{L}; \tag{43}$$

$$\bar{y}_{H} = c_{H}; \tag{44}$$

6. The factor markets clear:

$$n \int_{x_{1}}^{x_{2}} l^{NT} \left( y^{NT}(x) \right) dF(x) + n \int_{x_{1}}^{x_{2}} l^{I} \left( y_{L}(x) \right) dF(x) + n \int_{x_{1}}^{x_{2}} l^{\mu}_{\mu} \left( y_{\mu}^{NT}(x) \right) dF(x) +$$

$$n \int_{x_{1}}^{x_{2}} l^{\mu} \left( y_{H}(x) \right) dF(x) + n \int_{x_{1}}^{x_{2}} l^{T} \left( y^{T}(x) \right) dF(x) = \tilde{l}; \tag{45}$$

7. The number of variety available for consumption is the number of varieties produced:
\[ n \int_{\frac{x^n}{x_1^n}} dF(x) + n \int_{\frac{x^n}{x_2^n}} dF(x) + n \int_{\frac{x^n}{x_1}} dF(x) + n \int_{\frac{x^n}{x_2}} dF(x) + n \int_{\frac{x^n}{x}} dF(x) = \bar{n}. \]  

5 Calibration and Numerical Results

In this section I examine the quantitative properties of the model. I consider the sensitivity of the results to alternative parameterizations of the model economy. I explore how the equilibrium properties of the model change with variations in the probability of detection, tax rate and penalty cost. The model predictions determine what parameter or combination of parameters has a larger impact on tax compliance behavior.

I choose Russia as a country of analysis to calibrate my model for the year of 1998. Here I will explain the methods I used for my parameterization.

I assume a Pareto distribution for firm productivity,

\[ f(x) = \begin{cases} \gamma x^{-\gamma - 1} & \text{for } x \geq 1 \\ 0 & \text{otherwise} \end{cases}, \]

where \( k = 1 \) is the minimum value for productivity \( x \geq k \); \( \gamma > 0 \) is a shape parameter that determines the skewness of the Pareto distribution. In addition to being tractable, this distribution provides a reasonable approximation of observed variation in firm productivity. It is difficult to calibrate the shape parameter \( \gamma \) since there is no direct data on firms’ productivity levels. In addition, we cannot recover the true distribution of productivity because of non-observability of output (typically we have data on sales). I use 1998 data on employment size across Russian firms using the dataset “Fundamental information across Russian firms” provided by ZAO “Russian Investment Association”. I follow Gabaix et al. (2003) and Luttmer (2003) to construct the size distribution of firms. Let \( S \) denote the actual employment size of the firm. I rank firms from largest (rank 1) to smallest (rank \( n \)) so that \( S_{(i)} \geq K \geq S_{(n)} \). I then draw a graph, known as Zipf’s plot: on the y-axis, the log of the rank (the largest firm has log rank \( \ln 1 \), the smallest firm has log rank \( \ln n \) ) takes the log of employment level\(^5\). I find that the size distribution of firms in Russia is different from the U.S. firm distribution. As in Luttmer (2003), the U.S. size

\(^5\) The given number of firms \( n \) in 1998 is equal to 5537. Hence, \( \ln 5537 = 8.6 \).
distribution of firms almost perfectly fits a straight line with the slope -1.06. The Russian micro-level dataset misses a number of small firms. They constitute sectors like wholesale and retail trade, health care and social work, education, art and culture, financial services, and public administration. The possible explanation is that such small firms do not submit their budget reports to avoid being seen and caught for tax evasion. This proves the fact that in Russia small firms evade more than large firms. Figure 1 shows the size distribution of firms generated by the firm-level data for 1998. To find a Pareto shape parameter, economists run a regression of the log of number of firms on the log of employment to obtain the tail index of the Pareto distribution, assuming the data is generated by a Pareto distribution. The regression takes a linear form

$$y = Xb + e,$$

where $X$ is an $n$-by-2 matrix, where the first column is an identity column vector, and the second column is a column of the log of employment; $y$ is an $n$-by-1 vector of the log of number of firms; the vector $b$ is a 2-by-1 matrix of regression coefficients; and the vector $e$ is an $n$-by-1 vector of the residuals. Due to the fact that there must be a number of missing firms, I assume that $\gamma$ is equal to 2.5. I have chosen it to be greater than 2 to make sure that $\gamma$ has a finite mean.

The wage rate is normalized to one. To calibrate the values for the entry cost $f$, I use the study by Djankov et al. (2002) who collected data on the start-up costs across firms in 85 countries in 1999. The data covers all procedures – the number of procedures, official time, and official cost – that a start-up firm must bear before it can operate legally. For example, in Russia an entrepreneur must complete 20 procedures which take at least 57 days and must pay $449 US dollars to open a business. This is equivalent to 11,076.80 rubles in 1999 and approximately equals to 5,467.14 in 1998 rubles. Barkhatova (2002) estimates that in 1997 in Russia the costs of formal registration of a company was approximately 4,804 rubles. This includes State registration of a small company that requires paying the following fees: 50% of the initial capital, the minimum amount of

---

6 The average ruble/US dollar exchange rate in 1999 is 24.67.
7 The formula for turning ruble figures from 1999 into 1998’s rubles is the following:

$$\text{Amount in 1998 rubles} = \text{Amount in 1999 rubles} \times \frac{\text{Price level in 1998}}{\text{Price level in 1999}} = 11,076.80 \times \frac{6,099}{12,357} = 5,467.14$$

I use http://ier.org.ua/tables/Table%202.1%20Q.pdf for CPI index.
which is 100 times the official minimum wage, that is $83.5 \times 100 = 8,350$ rubles, so at the very beginning it is necessary to pay half of this amount, that is, 4,175 rubles; registration fees to the municipal budget are 150 rubles; registration fees to the State Statistics Committee are 144 rubles; registration fees for specimen signatures (four signatures) are 355 rubles; thus state registration of a private company costs 4,804 rubles. This amount in 1998 dollars is equal to 5,262.14 rubles, which is roughly five times the average nominal monthly wage\(^8\). Both estimates are very close. Here I assume that a fixed entry cost $f = 5$ which is five times the average nominal wage rate.

The penalty cost for tax evasion in Russia in 1998 was a fine of 200-500 times the minimum wage. The minimum wage in Russia in 1998 was 83.49 rubles. The penalty cost of tax evasion is between 16,698 and 41,745. In calibration I am going to use the average of these two numbers (29,221.5) which is 350 times the minimum wage or 28 times the average nominal wage. Hence the penalty cost for tax evasion $\zeta$ is equal to 28.

To calibrate the effective tax rate $\tau$, I analyze statistical balance reports of 5537 firms in 1998. Table 2 gives a sample balance sheet of an industrial firm submitted on January 1, 1998. (I have included only the data relevant for my analysis.) The balance sheet contains an identification number of a firm (OKPO), gross and net wages, social security contributions and information on number of employees. All earned wages are subject to one-percent mandatory pension fund deductions. Using all the above information, I can calculate the tax rate in each firm as follows:

$$\text{tax} = \frac{(1 - 0.01)NZP^{\text{declared}} - RZP}{NZP^{\text{declared}}} \times 100\%, \quad (47)$$

where $NZP^{\text{declared}}$ is the nominal declared wage payments before deductions, and $RZP$ is the nominal wage payment after deductions.

To find the effective tax rate, that is, the tax rate paid by the firms that fulfill their tax obligations, I use two selection mechanisms. One mechanism is based on the difference between average monthly wages prescribed by Goskomstat across industries, $w'$, and wages paid by the firm, $w^f$. If the difference between these two, $w' - w^f$, is less than or equal to zero, then the firm fully pays its tax duties, otherwise, I consider it to be a tax

---

\(^8\) The average nominal monthly wage in 1998 was 1,051.50 rubles.
cheater. Another method is based on the assumption that all firms truly pay their pension fund contribution duties (since they care about their retirement) which constitutes 28 percent of employee’s wages. Knowing the data for pension fund contributions, we can find the true value for gross wages as follows:

\[ NZP^{\text{true}} = \frac{PFC}{0.28}. \]

The true value of payroll tax payments can now be found as

\[ (1 - 0.01)NZP^{\text{true}} - RZP. \]  \hspace{1cm} (48)

The size of tax evasion is then a difference between (39) and (38), that is

\[ (1 - 0.01)\left( NZP^{\text{true}} - NZP^{\text{declared}} \right) \]  \hspace{1cm} (49)

Equation (49) allows me to distinguish between tax payers and tax evaders. To calculate the effective tax rate, I select only tax payers and take the average payroll tax. I find that the efficient tax rate \( \tau = 14.62\% \).

In 1998 the total number of firms in Russia, denoted by \( n \) is assumed to be 2.9 million firms (see Table 3) and total active employment \( l \) is 58.437 million people. The data limitations seem to be a sufficiently challenging obstacle to my analysis, and, particularly, the empirical specification of the parameters \( \rho, \bar{y}_L \) and \( \bar{y}_H \). The parameter \( \rho \) governs the elasticity of substitution \( 1/ (1 - \rho) \), the willingness of buyers to substitute goods produced by taxpayers (legal goods) and goods made by tax evaders (informal goods) of the same variety. The parameter \( \rho \) is chosen to be 0.7 so that the condition \( \gamma > \sigma - 1 \) holds. The elasticity of substitution is now equal to 3.33. Note that when \( \sigma > 1 \), as in our case, a decrease in \( \rho \) means an increase in the informal sector. In our case, due to the higher elasticity of substitution, individuals are more willing to substitute the formal good for the informal good. The parameters \( \bar{y}_L \) and \( \bar{y}_H \) indicate the boundary levels of output – low and high, respectively - so that if the firm’s production level falls in-between these two values it is detected with probability \( 1 - \bar{\mu} \). I have not attempted to produce an econometric estimate of the parameters \( \bar{y}_L \) and \( \bar{y}_H \), I simply let
\(\bar{y}_L = 600\) and \(\bar{y}_H = 1200\) so that the informal share of output is close to 44 percent\(^9\). I summarize the calibrated parameter values in Table 4.

Next I do a sensitivity analysis, beginning with the baseline model. Assuming \(\bar{\mu} = 1\), we have a production function of the form given by Figure 2 and Figure 3. The figures show that the small firms continue evading taxes until they start to produce at \(\bar{y}_H\), that is, 1200 units (which corresponds to the firm’s labor efficiency level of 6.72 or nearly 183 workers employed). To avoid paying all taxes, they keep producing at this pre-determined level until they reach the efficiency of 9.76. Since the labor productivity grows, such firms no longer need to maintain the same number of workers, and as Figure 3 shows, the employment level declines to 129 (129 workers employed). The fact that firms need fewer employees means they can make lower payroll tax payments. Moreover, to avoid being seen, they charge the same price as low-efficiency firms. Figure 4 and Figure 5 illustrate this situation. A higher price and lower labor costs generate higher profit (see Figure 6). Since I let the tax rate be zero, the production, price and profit functions of the tax evaders with zero probability of detection and the production, price, and profit functions of the tax evaders with a \(1 – \bar{\mu}\)-chance of getting caught coincide.

Figure 7 illustrates the structure of firms in the baseline model. Tax evading firms constitute 15.54% of all the firms in the economy, tax paying firms 0.34%, and the remaining 84.12 % decide to exit the market due to high entry costs. Given the parameters of the model, the size of the informal economy is nearly 44.04%, close to empirical estimates of the size of the shadow economy in Russia.

Next, I vary the probability of tax detection to see how it affects firms’ compliance decision. Assume that the firms face a 2-percent chance of getting caught. Figure 8 shows that the small firms with efficiency levels between 2.09 and 5.46 produce less than the low threshold level \(\bar{y}_L\) and are not monitored. According to Figure 9, this corresponds to the firms with more than 16 but less than 114 workers. Once the firms reach a productivity level of 5.46 or higher, they can produce more than \(\bar{y}_L\) and decide to fix their production at \(\bar{y}_L\) pretending as if they were low-efficiency firms. A rise in efficiency

\(^9\) The IMF analysis shows that the size of shadow economy for Russia in 1998 is 44 percent.
level from 5.46 to 5.86 allows such firms to reduce the number of employees from 114 to 107 and to not pay any taxes. The firms with productivity greater than 5.86 (or 133 workers) decide that they would be better off producing more than $y_L$ and take a small risk of being caught. They continue to do so until the total number of workers equals 183 workers, and they are able to produce no less than $y_H$. Once firms are able to produce more than $y_H$, they face a similar dilemma: either to fix their production level at $y_H$ and have a small chance of being caught for tax evasion, or continue to produce more than $y_H$ and pay all taxes. As Figure 9 shows, a small fraction of such firms benefit more by fixing their output at $y_H$ while others with more than 272 workers decide to fully comply with their tax obligations and produce more than 2595 hundreds units of output. Figure 10 and Figure 11 show the price function as a function of efficiency and production level, respectively. Due to the fact that firms with $y_L$ and $y_H$ fix their prices (as shown in Figure 10 and Figure 11), they are able to significantly increase their profit, illustrated in Figure 12.

The structure of firms is slightly different from the baseline. We observe a small increase in tax evading firms from 15.54% to 15.56% of which 14.70% evade taxes with no risk and 0.86% face a 2-percent chance of being caught. The number of tax paying firms stays the same and is equal to 0.34%, and the remaining 84.10% decide to exit the market due to the given parameters. Given the parameters of the model, the size of the informal economy decreases and becomes 43.89% which is lower by 0.34%.

Finally, I vary penalty costs and tax rates together with the probability of tax detection to see their impact on the size of informal sector, tax revenues, and calibrated income. Certainly, zero taxes increases real income but the government needs money to finance purchases, so this would not be possible (see Table 5). The higher the probability of detection becomes, the harder it is for the firms to avoid being seen. If so, the tax revenue increases and the informal sector becomes lower (see Table 6). An increase in penalty costs provides an additional constraint for firms and makes tax evasion less desirable. As a result, the informal sector declines and tax revenues increase.
6 Conclusion

In this paper I considered the tax evasion decision of firms. To acknowledge the heterogeneity of plants, I introduce Ricardian differences in technological efficiency across producers. To explain the coexistence of tax evaders and taxpayers even within the same industry, I specify the governmental policies. Based on the tax policies, I identify efficiency levels of the firms that go underground and assume that mostly inefficient firms choose to operate in the shadow economy. I build a monopolistic competition model of tax evasion in which inefficient firms decide to evade taxes whereas efficient firms make all tax payments.

I calibrate the model to Russia in 1998 and find that to fight tax evasion the government needs to increase monitoring and/or set up higher fixed costs. Assuming that the total number of firms is fixed, this reduces the number of tax evaders and reduces the size of shadow economy. A large increase in the probability of detection and/or increase in penalty costs also lead to an increase in tax revenues.

The model can be further extended. Based on the size distribution of firms, it will be interesting to understand why some small firms decide to report their employment level while others decide to avoid being seen at all.
References


Appendices

Appendix A: Tables

Table 1 Percentage of Sales Reported to Tax Authorities, by Size of Firm

<table>
<thead>
<tr>
<th>Percentage of sales</th>
<th>Firms</th>
<th>100</th>
<th>90-99</th>
<th>80-89</th>
<th>70-79</th>
<th>60-69</th>
<th>50-59</th>
<th>&lt;50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td></td>
<td>25.83</td>
<td>10.30</td>
<td>9.78</td>
<td>8.05</td>
<td>5.38</td>
<td>8.17</td>
<td>11.48</td>
</tr>
<tr>
<td>Medium</td>
<td></td>
<td>36.05</td>
<td>11.68</td>
<td>9.30</td>
<td>6.72</td>
<td>5.06</td>
<td>6.52</td>
<td>7.94</td>
</tr>
<tr>
<td>Large</td>
<td></td>
<td>41.43</td>
<td>10.70</td>
<td>7.53</td>
<td>5.45</td>
<td>4.10</td>
<td>5.19</td>
<td>7.79</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>32.96</td>
<td>10.93</td>
<td>9.13</td>
<td>7.01</td>
<td>4.99</td>
<td>6.92</td>
<td>9.33</td>
</tr>
</tbody>
</table>

Total = average for all firms in the WBES sample.
Source: Batra et al. (2003)

Table 2 Statistical Report of an Individual Firm (in thousands), 01.01.1998

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>OKPO Number</td>
<td>OKPO</td>
<td>31129621</td>
</tr>
<tr>
<td>Gross Wages</td>
<td>NZP</td>
<td>6476430</td>
</tr>
<tr>
<td>Net Wages</td>
<td>RZP</td>
<td>4738270</td>
</tr>
<tr>
<td>Social Security Contributions:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Social Security Fund</td>
<td>SSFC</td>
<td>333062</td>
</tr>
<tr>
<td>Pension Fund</td>
<td>PFC</td>
<td>2241161</td>
</tr>
<tr>
<td>Employment Fund</td>
<td>EFC</td>
<td>93946</td>
</tr>
<tr>
<td>Medical Insurance Fund</td>
<td>MIFC</td>
<td>240726</td>
</tr>
<tr>
<td>Number of Employees</td>
<td>L</td>
<td>95</td>
</tr>
</tbody>
</table>

Source: ZAO “Russian investment corporation”
Table 3 Number of Small Businesses and Total Number of Firms in Russia (in thousands)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Firms</td>
<td>560.0</td>
<td>865.0</td>
<td>896.9</td>
<td>877.3</td>
<td>841.7</td>
<td>861.1</td>
<td>868.0</td>
</tr>
<tr>
<td>Total Number of Firms</td>
<td>609.0</td>
<td>1244.9</td>
<td>1946.3</td>
<td>2249.5</td>
<td>2504.5</td>
<td>2727.1</td>
<td>2901.2</td>
</tr>
<tr>
<td>% of Small Firms to Total Number of Firms</td>
<td>92.0</td>
<td>69.5</td>
<td>46.1</td>
<td>39.0</td>
<td>33.8</td>
<td>31.6</td>
<td>29.9</td>
</tr>
</tbody>
</table>


Table 4 Calibrated Parameters for Russia, 1998

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Data</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>Total number of firms</td>
<td>2901200</td>
<td>Goskomstat, 1998</td>
</tr>
<tr>
<td>$l$</td>
<td>Total labor supply</td>
<td>58437000</td>
<td>Goskomstat, 1998</td>
</tr>
<tr>
<td>$f'$</td>
<td>Entry fixed costs</td>
<td>5</td>
<td>Djankov et al. (2002)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Curvature parameter</td>
<td>0.7</td>
<td>The elasticity of substitution is 3.33</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Pareto distribution parameter</td>
<td>2.5</td>
<td>A tail index of the size distribution of firms</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Probability of not being detected</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Penalty costs</td>
<td>28</td>
<td>28 times the nominal wage</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Payroll tax rate</td>
<td>0.1462</td>
<td>The effective tax rates across tax payers</td>
</tr>
<tr>
<td>$\bar{y}_L$</td>
<td>Low level of output</td>
<td>600</td>
<td>To match the shadow economy</td>
</tr>
<tr>
<td>$\bar{y}_H$</td>
<td>High level of output</td>
<td>1200</td>
<td>To match the shadow economy</td>
</tr>
<tr>
<td>$w$</td>
<td>Wage rate</td>
<td>1</td>
<td>Normalization</td>
</tr>
</tbody>
</table>
### Table 5 Tax effect on calibrated income in Russia, 1998 (in millions)

<table>
<thead>
<tr>
<th>Probability of detection</th>
<th>$1 - \mu = 0$</th>
<th>$1 - \bar{\mu} = 0.02$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax rates, %</td>
<td>Real Income</td>
<td>Change, %</td>
</tr>
<tr>
<td>$\tau = 0.00$</td>
<td>91966</td>
<td>0.00</td>
</tr>
<tr>
<td>$\tau = 11.70$</td>
<td>89616</td>
<td>-2.55</td>
</tr>
<tr>
<td>$\tau = 13.16$</td>
<td>89266</td>
<td>-2.94</td>
</tr>
<tr>
<td>$\tau = 14.62$</td>
<td>88906</td>
<td>-3.33</td>
</tr>
<tr>
<td>$\tau = 16.08$</td>
<td>88535</td>
<td>-3.73</td>
</tr>
<tr>
<td>$\tau = 17.54$</td>
<td>88154</td>
<td>-4.15</td>
</tr>
</tbody>
</table>

Sources: Own calculations

### Table 6 Comparative Statics on Shadow Economy and Tax Revenues for the case of $1 - \bar{\mu} = 0.2$

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\tau = 0.15$</th>
<th>$\tau = 0.12$</th>
<th>$\tau = 0.18$</th>
<th>$\zeta = 28$</th>
<th>$\zeta = 22.4$</th>
<th>$\zeta = 33.6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change wrt to baseline (in %)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shadow Economy</td>
<td>-0.90</td>
<td>-1.83</td>
<td>20.96</td>
<td>-0.90</td>
<td>-0.72</td>
<td>-1.05</td>
</tr>
<tr>
<td>Tax Revenues</td>
<td>4.14</td>
<td>-13.96</td>
<td>-0.09</td>
<td>4.14</td>
<td>3.86</td>
<td>4.39</td>
</tr>
</tbody>
</table>

Sources: Own calculations
Appendix B: Figures

Figure 1 Size Distribution of Russian Firms in 1998
Source: Own calculations
\[ y_H = 1200, \tau = 0.1462, f = 5 \]

Figure 2 Production as a function of efficiency for the case of \( 1 - \bar{\mu} = 0 \).

\[ y_H = 1200, \tau = 0.1462, f = 5 \]

Figure 3 Production as a function of employment for the case of \( 1 - \bar{\mu} = 0 \).
Figure 4 Price as a function of efficiency for the case of $1 - \bar{\mu} = 0$.

Figure 5 Price as a function of production for the case of $1 - \bar{\mu} = 0$. 

$y_H = 1200$, $\tau = 0.1462$, $f = 5$
\[ yH = 1200, \tau = 0.1462, f = 5 \]

Figure 6 Profit as a function of production for the case of \( 1 - \bar{\mu} = 0 \).

Figure 7 The structure of firms for the case of \( 1 - \bar{\mu} = 0 \) (baseline)
y_L = 600, y_H = 1200, \tau = 0.1462, f = 5

Figure 8: Production as a function of efficiency for the case of $1 - \mu = 0.02$.

y_L = 600, y_H = 1200, \tau = 0.1462, f = 5

Figure 9: Production as a function of employment for the case of $1 - \mu = 0.02$. 
$y_L = 600, y_H = 1200, \tau = 0.1462, f = 5$

**Figure 10** Price as a function of efficiency for the case of $1 - \overline{\mu} = 0.02$.

$y_L = 600, y_H = 1200, \tau = 0.1462, f = 5$

**Figure 11** Price as a function of production for the case of $1 - \overline{\mu} = 0.02$. 
Figure 1: Figure 12 Profit as a function of production for the case of $1 - \bar{\mu} = 0.02$.  

$y_L = 600, y_H = 1200, \tau = 0.1462, f = 5$
Figure 13 Informal share of output for various levels of penalty costs $\zeta$ and probability of detection $1 - \overline{\mu}$.

$y_L = 600, y_H = 1200, \tau = 0.1462, f = 5$

Figure 14 Informal share of output for various levels of tax rates $\tau$ and probability of detection $1 - \overline{\mu}$.
$y_L = 600, y_H = 1200, \tau = 0.1462, f = 5$

Figure 15 Tax revenues for various levels of penalty costs $\zeta$ and probability of detection $1 - \bar{\mu}$.

$y_L = 600, y_H = 1200, f = 5$

Figure 16 Tax revenues for various levels of tax rates $\tau$ and probability of detection $1 - \bar{\mu}$.