Macroeconomics Qualifying Exam--303 Module
Claremont Graduate University
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Answer question 1 *and either* question 2 or 3. All parts are equally weighted.

1. Consider a single-good overlapping generations economy where agents live two periods in two different countries. Population in countries 1 and 2 grows geometrically at the same rate $n > -1$, so that $N^i_{t+1} = (1+n) N^i_t$, for $i=1,2$, where population N^i_0 may vary across countries, as can initial capital $K^i_0 > 0$. Assume initially that countries do not trade with each other (autarky).

a) Set up the consumer's optimization problem for country *i* at time *t* when leisure is not valued and agents work in the first period for firms and are retired in the second period of their lives. Define each variable that you use.

b) Find the time *t* optimality condition(s) for consumers. Set up and solve the profit maximization problem for a representative firm operating in a perfectly competitive market.

c) Define a competitive equilibrium in this two-country model in each country in per worker (i.e. per youngster) terms. State conditions that guarantee that the (autarkic) equilibrium in each country is unique.

d) Now permit each country to trade with each other, where goods are traded for capital, and labor is assumed immobile across countries. Let $U(c^i_0, c^i_1) = (1-\beta)\ln(c^i_0) + \beta \ln(c^i_1)$ and $F(K^i,N^i) = (K^i)^a (N^i)^{1-a}$ for $i=1,2$, with $0 < a < 1$, and assume that the depreciation rate $0 < \delta < 1$ is identical internationally. Assuming perfect (frictionless) capital markets, find the international capital market clearing condition in terms of the aggregate capital stocks. Show that frictionless markets cause the capital-labor ratio to equalize across countries.

e) If country 1 has three times the capital and population as country 2, find the steady state of this economy in terms of per capita capital in country 1. Prove that if initial capital per capita in country 1 is less than the steady state level, then for any time t, country 1 cannot be a capital borrower. To do this, find per capita foreign investment in country 1, $h^1 = H^1/N^1$, where H^1 is capital minus aggregate savings in country 1 as defined in the lecture notes. Find out when h^1 is positive, and then use the steady state per capita capital stock in country 1 to show that this can never occur. (Yes, this is hard). Tell me which country always exports consumption goods.

f) Draw a phase portrait showing growth in the two-country world economy using world per capita capital stock. With trade, does one country, or both countries have different growth rates with trade vs. the no trade equilibrium? Support your answer using the model.

g) Now consider Mexico's trade with the US by modifying the model in this problem. In this version of the model, Mexican consumers take as given the US interest factor RR which is, by assumption, constant at 1.06 (so that the US capital-labor ratio is constant). Savings in Mexico can either be invested locally, or can be invested in the US and earn return RR. Set up this model and find optimality conditons. Using the production and utility functions given in part (d) with $a=1/3$, show that Mexicans will initially chose in invest domestically, and when the Mexican capital-labor ratio becomes sufficiently high, then they will invest in the U.S. Find the Mexican capital-labor ratio at which Mexicans will begin to invest in the U.S.

2. Next, consider a one-country single-good overlapping generations economy where agents live two periods, working in the first period and retired in the second period. Leisure is not valued and population grows geometrically at rate $n > -1$. This model includes firms and productive capital.

a) Set up the consumer and the firm optimization problems. Find optimality conditions for generic (nonspecific) utility and production functions. Using one sentence for each condition, discuss the meaning of these optimality conditions.

b) Let production be Cobb-Douglas, $Y_t = AK_t^a N_t^{1-a}$, for some value of $A > 1$ which indicates aggregate productivity, K is aggregate capital and N is the number of young agents. Suppose that agent's preferences result in the following savings function, $s_t = \beta w_t / R_{t+1}$, for $0 < a(1+n)/(1-a) < \beta < 1$. Draw a phase portrait in the space of the current period's, and next period's per capita capital when depreciation $\delta=1$. Indicate with arrows of motion the dynamics of this economy.

c) Interpret the dynamics of the model above relative to other growth models you have studied in this class. What does the model in part (b) say about steady states and their stability? Show what happens to the phase portrait when $\beta < a(1+n)/(1-a)$. Discuss what this means and identify the reason why the dynamics have changed relative to those in part (b).

d) Suppose that a developing country's per capita saving function is the one given in part (b). *What does this model predict for this country's ability to catch-up to US* living standards? Does the international evidence support such a view? Using your answers to these questions, argue that the model in part (b) is either: i) a good model of developing countries; ii) a good model for developed countries; iii) a good model of both developing and developed countries; iv) a bad model for both types of countries. Be clear on your reasoning and limit your answer to ½ page.

3. Consider a Cass-Koopmans growth model with an infinitely-lived representative agent. Let lifetime utility be given by $\Sigma_{t=0}^{\infty} \beta^t U(c_t)$, where U(c) satisfies the Inada conditions, $\beta \in (0,1)$ is the subjective discount factor and c is consumption. Assume population is growing geometrically, $N_{t+1} = (1+n)N_t$. Let aggregate output be given by $Y_t = AF(K_t, N_t)$, where A is a positive constant, K is capital and F(K, N) satisfies the Inada conditions. Let $\delta \in [0,1]$ be the depreciation rate of capital.

a) Describe what is meant by a "planning problem." Write down a such a problem in per capita terms and describe each aspect of it completely but briefly.

b) What are the choice and state variables? Find the planner's optimality condition. Prove that this condition is equivalent to a market equilibrium (carefully!). What is the meaning of the transversality condition?

c) How many variables have dynamics in this problem? Find all steady states. Derive the phase portrait by partitioning the phase-space into areas where the variables with dynamics increase or decrease. Include arrows of motion.

d) Now suppose that the per capita production function is $y_t = A_t k_t^a$, where $A_t = k_t^\gamma$ for $\gamma>0$, is a technological spillover from the invention of new technology. Determine how the value of γ affects the dynamics of this problem by drawing different phase portraits as γ varies. Which phase portrait is most plausible given all you know about US data?