Macroeconomics Qualifying Exam -- 303 Module
Professor Paul J. Zak
Claremont Graduate University
February, 2005

Please answer question 1 and **either** question 2 **or** question 3. Each subpart is worth 10 points.

Q1. In honor of the Kydland-Prescott Nobel Prize this year, this question analyzes a Cass-Koopmans model with an infinitely lived representative agent who obtains utility from consumption, *c*, and leisure, ℓ. The model is given by

$$Max_{ct,\,\ell t} \sum_{t=0}^{\infty} \beta^t [U(c_{,t}) + V(\ell_t)]$$

s.t.

$$c_t + i_t = f(k_t, h_t)(1-\tau) + \sigma_t$$
$$k_{t+1} = (1-\delta)k_t + i_t$$
$$h_t + \ell_t = 1,$$

where *k* is capital, *h* is labor hours, $\delta \in [0,1]$ is depreciation, $\beta \in (0,1)$ is the agent's discount factor, $\tau \in [0,1)$ is the tax rate, and *i* is investment. Tax revenue is rebated back to agents through a lump-sum transfer σ. $U(\bullet)$, $V(\bullet)$ are neoclassical utility functions, and $f(\bullet)$ is a neoclassical production function: all have the standard properties and satisfy the Inada conditions. There is no population growth nor any stochastic variables.

a) Is this a competitive or planning model? Support your answer.

b) Prove or disprove: this model is equivalent to a competitive/planning model with similar constraints. Be sure to write down the "other" type of model your proof is based upon.

c) State the government budget constraint. Prove or disprove: when $\tau>0$ the planner optimally directs the agent to work less time than when $\tau=0$.

d) Find the FOCs of the model.

e) State the complete set of optimality conditions for the model.

f) *Derive* the phase portrait for the model. Identify all steady states and their stability properties. Show all your work.

g) Following a recent publication, now let the tax rate τ be stochastic, $\tau \sim G$, for some CDF *G* with finite moments. Is τ a state variable (support your answer)? Write down the model with the stochastic tax rate.

h) Prove or disprove for $\tau \sim G$: when $\tau>0$ the planner optimally directs the agent to work less time than when $\tau=0$

i) In the aforementioned stochastic tax paper, the authors said they were “agnostic” (i.e. don’t care) as to the source of shocks to the economy. Use this statement to critique the Kydland-Prescott real business cycle model. First, state the standard version of the Kydland-Prescott real business cycle model and compare it to the stochastic tax model you analyzed. For example, what is its explanatory value of the standard model? What is/0are the mechanism(s) that cause business cycles in the model? Do these have empirical support? (do this in 1 blue book page or less)

j) In 1 blue book page or less, fully describe the policy implications of the standard Kydland-Prescott real business cycle model and the stochastic tax version of the model and discuss why these are “good” policy for the economy.

Q2. Consider a two period life OLG model in which parents care about their children's human capital. Let h_t be the human capital of an agent born at time t. Population is constant and normalized to unity. Then, writing all terms relative to human capital, an agent born at time t solves the following optimization problem,

$$Max\ (1-\beta)\ ln(c_{0,t}) + \beta\ ln(c_{1,t+1}) + \gamma\ ln(h_{t+1})$$

s.t.

$$c_{0,t} = w_t h_t - \kappa - s_t$$
$$c_{1,t+1} = R_{t+1} s_t$$
$$h_{t+1} = \omega h_t \kappa^{\eta},$$

where $\beta \in (0,1)$ is the agent's subjective discount on consumption utility, $\gamma > 0$ is the weight parents place on their children's human capital, $\kappa \geq 0$ is the tax parents pay to fund public schools, c_i is consumption in period $i=0,1$ of an agent's life, w is the wage, R_j is the interest factor from period j to period $j+1$, s is savings, $\omega>0$ is the human capital inherited from one's parents, and $\eta \in (0,1)$ is the effectiveness of public schools in raising human capital.

a) Identify the state and choice variables at time t for this model.

b) Let output Y be produced with physical capital, K, and aggregate human capital, H, $Y=K^{\alpha} H^{1-\alpha}$, for $\alpha \in (0,1)$. Solve the representative firm's profit maximization problem for w_t and R_{t+1}.

c) Write down the equilibrium dynamical system for this model in terms of the state variable(s). Completely and carefully define a competitive equilibrium for this model.

d) Produce a phase portrait of this economy's dynamics, including deriving arrows of motion.

e) Prove or disprove: for some $\kappa > 0$ there exists a poverty trap for this model. Using your proof/disproof, discuss the role public education at promoting or inhibiting growth.

Q3. Consider a two period life OLG model with production. Population is constant and normalized to 1 and consumers are retired in old age. A consumer who has logarithmic utility solves the following utility maximization problem,

$$\begin{aligned} &\textit{Max } (1-\beta)\, ln(c_{0,t}) + \beta\, ln(c_{1,t+1}) \\ &\textit{s.t.} \\ &c_{0,t} = w_t - s_t \\ &c_{1,t+1} = R_{t+1}\, s_t \end{aligned}$$

where $\beta \in (0,1)$ is the agent's discount factor, c is consumption, and R is the yield on savings, s.

a) Find the optimal savings s^*. Derive a condition using a general neoclassical production function $f(k)$ that satisfies is strictly increasing, continuous, and concave in capital, k, that guarantees that s^* is concave in k.

b) Let the wage be $w_t = \alpha A k_t$, and $r_t = (1-\alpha) A k_t$, for $\alpha \in (0,1)$ and $A>0$. Use these prices to construct the capital market clearing condition in terms of current and future state variables. Find all steady states.

c) *Derive* the phase portrait, including arrows of motion. Identify the stability properties of all steady states.

d) Prove or disprove: This economy exhibits perpetual growth.

e) We now modify the model so that $w_t = \alpha k_t$, and $r_t = (1-\alpha) k_t$, for $\alpha \in (0,1)$. Rederive the phase portrait and discuss in 1 bluebook page or less what drives the dynamics of this version of model as compared to the previous version.