Macroeconomics Qualifying Exam – 303 Module
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Answer 2 of the 3 questions. All parts are equally weighted.

1. Consider a Solow growth model in which population is growing geometrically, $N_{t+1} = (1+n)N_t$, $n > -1$ and capital depreciates at rate $\delta \in [0,1]$.

a) Write down the model in per capita terms, defining each variable you use briefly. Where does this equation come from?

b) Find all steady states and use them to determine the dynamics of the model. Illustrate the dynamics by drawing the phase portrait with arrows of motion. What assumptions did you use in drawing the phase portrait?

c) Define the "golden rule". Find the golden rule level of capital.

d) Prove that, under some conditions that you will find, the golden rule level of capital exceeds the steady state value. Discuss the conditions for which the golden rule coincides with the steady state.

2. Consider a Cass-Koopmans optimal growth model in which agents are infinitely lived and are partitioned into 2 household types, with $i \in \{1,2\}$ denoting the type. Each household type earns a different wage, ω^i and does not value leisure. Assume that population is constant. Denote the number of type i individuals as N^i with total population given by $N = N^1 + N^2$.

a) Write down the lifetime utility maximization problem for a type i agent living in such an economy. Define all variables and parameters that you use. Derive the optimality conditions for consumers.

b) How many markets are there in this model? Define a competitive equilibrium.

c) Let

$$F(K_t, N) = K_t^\alpha[(N^1)^{1-\alpha} + (N^2)^{1-\alpha}]$$

for $\alpha \in (0,1)$. Solve the firm's problem to determine the interest rate and the wage rates for $i = 1,2$. Find all steady states for this economy explicitly.

d) Suppose now that the economy is at a steady state and unexpectedly a lump-sum tax, τ, is imposed on type 1 agents while type 2 agents are untaxed. The proceeds of this tax are dumped into the ocean somewhere off New Jersey. Using graphs, explain what will happen to the consumption of each agent, the interest rate and output.

3. Consider an overlapping generations model with productive capital and government debt

where agents live two periods and solve

$$Max_{c_{0,t},c_{1,t+1}} U(c_{0,t}, c_{1,t+1})$$

s.t.

$$\begin{aligned} c_{0,t} &= w_t - s_t \\ c_{1,t+1} &= R_{t+1} s_t \end{aligned}$$

where c_i is consumption at age $i = 0, 1$, t is time, w is wage, N is population which is assumed constant, s is savings, $R = 1 + r - \delta$ is the net return on savings, where $\delta \in [0, 1]$ is the depreciation rate on capital. Assume that the following functional forms hold,

$$U(c_0, c_1) = c_0^{1-\beta} c_1^{\beta},$$

and

$$F(K, N) = K^{\alpha} N^{1-\alpha}$$

for $\alpha, \beta \in (0, 1)$. In addition, there is a government that functions solely to roll over its debt in perpetuity using the budget constraint

$$B_{t+1} = R_t B_t$$

where B is the aggregate stock of government debt which has the same return as physical capital.

a) Derive consumer and firm optimization conditions. What is the asset market clearing condition in per capita terms? Interpret this condition. What is/are the state variable(s)?

b) How many stationary equilibria are there in this model? Identify each one.

c) Draw the phase portrait and determine the dynamics of the system.

d) Suppose that due to a war with Canada, the U.S. government is required to sell \$50 billion of bonds. The government will pay interest on this debt in perpetuity by adding it to the existing stock of debt. Demonstrate (mathematically and/or graphically) and explain the effect on the economy of this increase in the level of debt. Describe the dynamics of this process.