Macroeconomics Qualifying Exam – 303 Module
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Answer question number **one** and then **either** question two or three. That is, you will answer a total of **two** of the following three questions which must include question number one. Put **each answer** in a separate blue book with the question number and your assigned exam number on it.

1. Consider a discrete-time optimal growth model with an infinitely lived representative agent in which a country sells a natural resource, denoted z, on the world market (this question is a variant of the model of Rodríguez & Sachs (1999), Journal of Economic Growth, pp277-303),

$$\underset{c_t}{Max}\ \Sigma_{t=0}^{\infty}\beta^t \ln(c_t)$$

s.t.

$$\begin{aligned} y_t &= Ak_t^{\alpha} \\ c_t &= y_t - i_t^d \\ k_{t+1} &= (1-\delta)k_t + i_t^d + i_t^m. \end{aligned}$$

The first constraint defines the production process where k is the domestic capital stock, for $\alpha \in (0,1)$ and $A > 0$. The second shows that output can be expended on either consumption, c or domestic investment, i^d. The third constraint is the stock accounting equation for the (domestic) capital stock, which includes foreign investment i^m. Population is assumed constant and normalized to unity, $\delta \in [0,1]$ is the depreciation rate of capital, and $\beta \in (0,1)$ is the discount factor. Foreign investment is funded by selling the natural resource outside the country, with the balance of payments holding at equality each period,

$$z_t = i_t^d.$$

ASSUMPTION 1. Resource extraction is a fixed portion of output, $z_t = \eta y_t$, for $\eta \in (0,1)$.

a) Define a planning problem and a market problem generically. Prove that the model above is either a planning problem or a market problem.

b) Solve for the Euler equation. State the complete set of optimality conditions for this model.

c) Find all steady state equilibria. Prove or disprove: The variable z is a state variable.

d) Draw a phase portrait and derive arrows of motion.

e) In a maximum of one blue book page, analyze the effect that natural resource extraction has on this economy, especially noting growth and welfare.

For the remainder of this problem, the assumption below replaces Assumption 1.

ASSUMPTION 2. Resource extraction is $z_t = Re^{-\lambda t}$, for $R, \lambda > 0$.

f) Examine what Assumption 2 does to the dynamics of this model. In particular, Prove or disprove: Assumption 2 causes the number of steady states to change. There is a difficulty drawing a phase portrait for this model: why?

g) Consider two economies, identical until time t_0, at which time country 2 discovers a natural resource. Use the results of part (f) to graphically demonstrate the following proposition: For a sufficiently high R, the economy with natural resources that starts out with a capital stock lower than its steady state will experience a consumption boom such that there exists a $T > t_0$ such that for all $t > T$, consumption exceeds steady state consumption in the economy without natural resources.

h) Explain in one blue book page the economic mechanisms that cause overconsumption to occur. This answer must be C^3: clear, careful, and complete.

2. Consider an overlapping generations model with productive capital where agents live two periods and solve

$$Max_{c^j_{0,t}, c^j_{1,t+1}} U(c^j_{0,t}, c^j_{1,t+1})$$

s.t.

$$\begin{aligned} c_{0,t} &= w^j_t - s^j_t \\ c_{1,t+1} &= R_{t+1} s^j_t \end{aligned}$$

where c^j_i is consumption at age $i = 0, 1$, for agents of type $j = A, B$. In addition, t is time, w^j is wage, s^j is savings, $R = 1 + r - \delta$ is the net return on savings, where $\delta \in [0, 1]$ is the depreciation rate on capital. Agents differ by their human capital and therefore earn different wages. Also, assume that population growth for each type of agent is $N^j_{t+1} = (1 + n^j)N^j_t$ with N^j_0 normalized to one $\forall j$.

a) What is/are the state variable(s)? Derive consumer optimality conditions.

b) This economy has a large number of firms operating in a perfectly competitive environment. The aggregate production function is given by $Y_t = K^\alpha_t[(N^1_t)^{1-\alpha} + (N^2_t)^{1-\alpha}]$. Solve for w^j and R. Prove or disprove: The given production function is *not* constant returns to scale.

c) How many markets are there in this model? Carefully and completely define a competitive equilibrium. Does your answer in (b) regarding the returns to scale of the production function affect your answer?

d) Now assume that $n^1 = n^2$ and that the for all times t, B-type human capital agents are a proportion $\beta \in (\frac{1}{2}, 1)$ of the total working population, $N_t = N^1_t + N^2_t$. That is, B-type agents are more numerous than type A agents. Write the capital market clearing condition in per worker terms.

e) State a theorem guaranteeing that there is a single interior steady state in this model. Assuming this theorem holds, find all steady states.

f) Draw the phase portrait assuming the theorem in part (e) holds, and include arrows of motion showing the dynamics of the system. State the stability properties of each steady state.

g) Determine the effect that a larger or smaller less-skilled population (i.e. a variation in β) has on this economy. Breifly discuss the policy implications of your results, especially regarding the distribution of income.

3. Consider a discrete-time Solow model with taxes and government investment. Let $\tau \in (0,1)$ be a proportional tax on aggregate income, and λ be government investment. In this model, savings comes from after-tax income, where production is Cobb-Douglas, $F(K,\lambda) = K^{\alpha}\lambda^{1-\alpha}$, where K is private capital. Population is constant and is normalized to 1.

a) Write down the government budget constraint when tax revenue is only and completely used to fund government investment. Note that government investment fully depreciates every period.

b) Next, construct the capital market equilibrium condition. Describe what this condition means (and define all terms you use) briefly.

c) How many steady states are there in this model? Draw a phase portrait illustrating your answer and include arrows of motion.

d) Prove or disprove: There exists a single value of τ such that there is a unique interior steady state for this model. Discuss the intuition for this result (or nonresult as the case may be) and draw a phase portrait showing the dynamic effects of an increase in the tax.

e) Determine the dynamics of this economy when the government chooses public investment and taxes optimally. Suppose that the government's objective is to maximize capital deepening, i.e. $Max_{\tau,\lambda}\frac{K_{t+1}}{K_t}$. This maximization is subject to two constraints, the government budget relation you stated in part (a), and the capital market clearing condition you constructed in part (b). Solve for the optimal λ and τ. (This model is a special case of Ghate & Zak (1999) which is downloadable from the working paper link on my webpage.)

f) Embed the optimal λ and τ into the capital market clearing condition. Draw a phase portrait for this economy and derive arrows of motion.

g) Prove that if $s > \frac{\delta(1-\alpha)^{\frac{\alpha-1}{\alpha}}}{\alpha}$, then growth in this economy continues forever without reaching a steady state. Briefly explain why this outcome occurs.