Macroeconomics Qualifying Exam – 303 Module

Professor Paul J. Zak

Claremont Graduate University

January, 2001

Answer 1 and **either** questions 2 or 3. All parts are equally weighted.

1. Consider a Solow growth model with population growth at rate $n > -1$, $N_{t+1} = (1+n)N_t$, where N_t is population at time t, savings rate $s \in (0,1)$, and depreciation rate $\delta \in [0,1]$. Let output at time t be given by the neoclassical production function $Y_t = F(K_t, N_t)$ which has the standard properties and satisfies the Inada conditions. This version of the Solow model includes lump-sum taxes of $\tau > 0$ per person. Importantly, savings is done from after-tax income. Tax revenue is spent on government consumption which is nonproductive.

a) Write down the capital market clearing condition in terms of aggregate capital stock K.

b) Use part (a) to derive the per capita capital market clearing condition showing each step in your derivation completely. Denote per capita capital stock at $k_t = \frac{K_t}{N_t}$.

c) Use part (b) to produce a phase portrait of this economy's dynamics. Include (but don't derive) arrows of motion.

d) Prove or disprove: This model has a unique interior steady state.

e) Now consider two economies described by the Solow model in per capita terms. Country A has savings rate $s_A > 0$ and lump-sum tax $\tau > 0$. Country B has savings rate s_B, with $s_A > s_B$, but there are no taxes in country B. Assume the production functions, depreciation rates, and rates of population growth are identical in both countries. Write down the capital market clearing condition for country A and for country B when both are closed-economies (i.e. there is no trade between them).

f) Using part (e) derive a condition relating savings rates s_A, s_B, and τ (and possibly other terms) under which growth is faster in A than in B. [Hint: Start with identical values of k_t in both countries.]

g) Interpret the condition you derived in part (f): what does it mean? when is it satisfied? when does it fail?

h) Use the information in (g) to draw a phase portrait for both countries in the same graph. Prove of disprove: Country B has a higher level of steady state capital stock than country A. (You will need the results in part (f) for this).

2. Consider a standard two period life overlapping generations model. The number of youngsters in this economy evolves according to $N_{t+1} = (1+n)N_t$, where N_t is the number of youngsters at time t, and $n > -1$.

A consumer born at time t solves the following utility maximization problem,

$$Max_{c_{0,t},c_{1,t+1}} U(c_{0,t}, c_{1,t+1})$$

s.t.

$$
\begin{aligned}
c_{0,t} &= w_t - s_t \\
c_{1,t+1} &= R_{t+1} s_t.
\end{aligned}
$$

a) What is/are the state variable(s) at time t? Find the first order condition for this model.

b) Construct the capital market equilibrium condition in per youngster terms using only the state variable(s) and constants using the equilibrium wage and interest rate from the firm's problem (which you need to set up and solve).

c) Completely and carefully define an equilibrium for this model.

d) Prove or disprove: This model has multiple interior steady states.

e) Produce (but don't derive) a phase portrait taking into account your answer to (d). Identify the stability properties of all steady states and include arrows of motion.

f) Now consider the impact of a contraction in the population, i.e. $n \in (0, -1)$. Consider two economies, one with $n > 0$ and one with $n < 0$. Prove or disprove: The country with $n < 0$ necessarily has higher per capita capital in the steady state than the country with $n > 0$. [Hint: Consider your result in part (d).]

3. Consider an optimal growth model with a proportional tax $\tau \in (0, 1)$ on wages. The government collects taxes from individuals and rebates the revenue to individuals as a lump-sum transfer, σ. Population is constant and is normalized to unity, and leisure is not valued.

a) Write down the market version of the optimal growth model with the tax and transfers included as specified above.

b) What is/are the state variable(s) for this model? What is/are the choice variables?

c) Find the Euler equation for the model in (a).

d) State the complete set of optimality conditions for this model.

e) Write down the planning version of this model.

f) Prove that the equilibrium for the market model in (a) and the planning model in (e) are identical. [Hint: Use the government budget constraint.]