Macroeconomics Qualifying Exam – 303 Module
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Answer 1 and either questions 2 or 3. All questions are equally weighted.

1. Consider a three period life overlapping generations endowment economy. The number of youngsters in this economy evolves according to $N_{t+1} = (1+n)N_t$, where N_t is the number of youngsters at time t, and $n > -1$.

A consumer born at time t solves the following utility maximization problem,

$$Max_{c_{0,t},c_{1,t+1},c_{2,t+2}} \ln(c_{0,t}) + \beta \ln(c_{1,t+1}) + \beta^2 \ln(c_{2,t+2})$$

s.t.

$$\begin{aligned} c_{0,t} &= e_0 - s_{0,t} \\ c_{1,t+1} &= e_1 + R_{t+1}s_{0,t} - s_{2,t+1} \\ c_{2,t+2} &= R_{t+2}s_{2,t+1}, \end{aligned}$$

where $\beta \in (0,1)$ is the agent's subjective discount on utility flows, c_i is consumption in period $i = 0,1,2$ of an agent's life, t is time, $e_i > 0$ is the endowment received in period i, R_j is the interest factor from period j to period $j+1$, and s_i is savings in period i.

a) Take the first order conditions for this model and solve for $s^\star_{0,t}$ and $s^\star_{2,t+1}$ in terms of parameters and interest factors. [Note: this is rather long.]

b) Prove of disprove: For any $R > 0$, optimal savings in middle age $s^\star_{2,t+1}$ is strictly increasing in e_1 and strictly decreasing in e_0.

c) Write down a "general" aggregate loan market equilibrium condition at time t (by general I mean for some nonspecific optimal savings relations). Be careful to get the timing (i.e. the subscripts) correct. Now write this in per youngster terms at time t.

d) Define a competitive equilibrium for this model.

e) Substitute in the savings relations you found in part (a) into the per youngster loan market condition in part (c). Simplify your answer as much as possible.

f) Discuss briefly (max=1 blue book page): Is the loan market in this economy functioning? That is, are loans actually being made or is the equilibrium autarkic? If loans are being made, provide a general condition that identifies lenders and borrowers.

g) Using (e), derive a condition under which the sequence of interest factors in this economy, $\{R_t\}_{t=1}^{\infty}$ are strictly decreasing.

h) Assuming the condition you found in (g) holds, interpret how individuals' behaviors and therefore the equilibrium in this economy change when $R_t = -1 \; \forall t > T$. Specifically, discuss what happens to the generational structure of model. [Hint: think about the biology of these agents].

2. Consider a two period life OLG model in which parents care about their children's human capital. Let h_t be the human capital of an agent born at time t. Population is constant and normalized to unity. Then, writing all terms relative to human capital, an agent born at time t solves the following optimization problem (assuming utility is logarithmic),

$$Max_{c_{0,t},c_{1,t+1},h_{t+1}}(1-\beta)\ln(c_{0,t})+\beta\ln(c_{1,t+1})+\gamma\ln(h_{t+1})$$

s.t.

$$\begin{aligned} c_{0,t} &= w_t h_t(1-\sigma)-s_t \\ c_{1,t+1} &= R_{t+1}s_t \\ h_{t+1} &= \omega h_t \sigma^{\eta}, \end{aligned}$$

where $\beta > 0$ is the agent's subjective discount on consumption utility, γ is the weight parents place on their children's human capital, $\sigma \in (0,1)$ is the parents' investment in their children's human capital, c_i is consumption in period $i = 0,1$ of an agent's life, t is time, w is the wage, R_j is the interest factor from period j to period $j+1$, and s is savings, $\omega > 0$ is the human capital inherited from one's parents, and $\eta \in (0,1)$ is the effectiveness of resources in raising human capital.

a) Identify the state and choice variables for this model.

b) Solve for the optimal investment in human capital, $\sigma^{\star}$ and the optimal savings function $s^{\star}$. Be sure to substitute out σ from $s^{\star}$ using $\sigma^{\star}$.

c) Let output Y be produced with physical and human capital, $Y = K^{\alpha}H^{1-\alpha}$, for $\alpha \in (0,1)$. Solve the representative firm's profit maximization problem for w_t and R_{t+1}.

d) Using (b) and (c), write down the equilibrium dynamical system for this model in terms of the state variable(s).

e) Produce a phase portrait of this economy's dynamics, including deriving arrows of motion.

f) Prove or disprove: This model has three steady states.

3. This question asks you to compare the dynamics of a standard two period overlapping generations model and standard Cass-Koopmans model. By standard I mean with a single asset, physical capital, used in production; for the OLG model having agents live two periods and being retired when old, with agents in a generation being homogeneous; and for the Cass model having an infinitely-lived representative agent. There are no taxes or transfers. Population is constant and is normalized to unity in both models, and leisure is not valued. "Proofs" in this question are equally valid if you use the mathematical equations describing the model, or if you use the geometry of the phase portraits.

a) For general utility and production functions that are both strictly increasing, continuous, and concave, and satisfy their respective Inada conditions, write down (or derive and then write down) the capital market clearing conditions for both models. You may use the planning version of the Cass model.

b) *Fully characterize* the dynamics of these models by producing phase portraits. [You can either derive these phase portraits or simply write them down.] Please include arrows of motion and identify steady states.

c) Prove or disprove for both models: there is a unique interior steady state.

d) Prove or disprove for both models: The equilibrium dynamics require that poor countries grow faster than rich countries.

e) Prove or disprove for both models: There must be an odd number of steady state equilibria.

f) Prove or disprove for both models: The inclusion of human capital always produces dynamics with perpetual growth.