Macroeconomics Qualifying Exam – 303 Module
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Answer 1 and **either** question 2 or 3. All questions are equally weighted.

1. Consider a Cass-Koopmans model with physical capital and valued leisure. Population is constant and is normalized to unity. In this model the infinitely-lived representative agent values leisure. Time is normalized to unity.

a) For general utility and production functions that are both strictly increasing, continuous, and concave, and satisfy their respective Inada conditions, write down the planning version of the model. Be sure to define all variables that you use. Let the utility from consuming goods and consuming leisure be separable.

b) Identify the state and choice variables. Define a "planning problem" and a "competitive problem" in general.

c) Produce the optimality condition(s) for the planner. Identify the optimal variable (with a "$\star$") that each condition implicitly defines.

d) Carefully state the complete set of conditions required for an optimum to obtain for this problem.

e) Derive the dynamical behavior of this model by producing phase portraits. Please identify all steady states and *derive* arrows of motion.

2. Consider a standard two period life overlapping generations "real business cycle" model with a random technology shock A_t, for all times t. Unlike the standard model, assume that the technology level A_t is **not** known for certain at time t (though A_{t-1} is known at time t), but A_t must be forecasted at using an autoregressive form

$$A_t = \rho A_{t-1} + \epsilon_t$$

where ϵ_t is a normally distributed with unconditional expected value 0, and $\rho \in (0,1)$. The population grows geometrically, $N_{t+1} = (1+n)N_t$, where N_t is the number of youngsters at time t, and $n \geq -1$. There is also an income tax $\tau \in [0,1]$ on youngsters.

A consumer born at time t who has logarithmic utility solves the following expected utility maximization problem,

$$Max_{c_{0,t},c_{1,t+1}} E_t(1-\beta)\ln(c_{0,t}) + \beta\ln(c_{1,t+1})$$

s.t. P. 2

$$c_{0,t} = w_t(1-\tau) - s_t$$
$$c_{1,t+1} = R_{t+1}s_t.$$

a) What is/are the state variable(s) for this problem? Write down the optimal savings function s_t^* that is the consumer's optimum for the problem above.

b) Construct the capital market equilibrium condition in per youngster terms using only the state variable(s) and constants. To do this, let the wage be $w_t = (1-\alpha)e^{A_t}k_t^{\alpha}$, where $k_t \equiv \frac{K_t}{N_t}$. Then, draw a graph of the equilibrium dynamics in this model in k_t, k_{t+1} space when technology is a random variable and follows the autoregressive expression given above. Include a 45 degree line and arrows of motion showing how the economy evolves. What does the persistence of a technology shock mean for the persistence of variations in capital, output and consumption?

c) Describe what happens the economy at time t if in the news it is reported that the "experts" expect that next period's value of A_t will be negative and large because it has been positive for so long, and most people believe this is true. Be explicit about causation, and effects over time.

d) President Bush has just added you to his Council of Economic Advisors to tell him how to get the economy out of the recession. Using the model above, offer the most effective policy to stimulate economic output and describe to the President how this would produce the desired effect.

3. Consider a two period life overlapping generations model *without* production and instead with an endowment $e_0 > 0$ for all youngsters (and no endowment for oldsters). Let population be constant and normalized to 1. Consider a model with a single asset, bonds, which have a 1 period maturity (i.e. pay off in 1 period). There a fixed supply $b > 0$ of bonds. The price of a bond at time t is q_t, and the following period it pays 1 unit of the consumption good. (This is called a "discount bond" as the price you pay is a discount from the principal thus providing an implicit rate of return; an example is T-bills.) Then the yield on a bond, $R_{t+1}^b = \frac{1}{q_t}$. Define $\gamma \geq 0$ as the quantity of bonds purchases (need not be an integer). The model below can be used to price bonds.

$$Max_{c_{0,t},c_{1,t+1}}(1-\beta)\ln(c_{0,t}) + \beta\ln(c_{1,t+1})$$

s.t.

$$c_{0,t} = e_0 - q_t\gamma$$
$$c_{1,t+1} = \gamma.$$

a) Find the first order condition by substituting in the constraints and differentiating with respect to γ. Solve for the optimal γ^*.

b) How does the demand for bonds γ^* change as the price q_t changes?

c) Using the demand for bonds equation from (a), impose an equilibrium by setting $\gamma^* = b$. Now use the equilibrium relationship to solve for the return on bonds R^b_{t+1} (using the definition above). How does R^b_{t+1} change when the endowment e_0 and patience β change? Explain why (i.e. behaviorally–what people are doing in markets) this occurs in each case.

d) We now modify the model to include both the bond and "inside money" loans with return R. Let δ be the quantity of inside money loans the agent makes. The model is now:

$$Max_{c_{0,t},c_{1,t+1}}(1-\beta)\ln(c_{0,t})+\beta\ln(c_{1,t+1})$$

s.t.

$$\begin{aligned} c_{0,t} &= e_0 - q_t\gamma - \delta \\ c_{1,t+1} &= \gamma + \delta R_{t+1}. \end{aligned}$$

Solving the model above, find: *i)* the relationship between R_{t+} and R^b_{t+1}; and *ii)* the equilibrium number of inside money loans made.

d) Now, modify the model so that the single asset is a one-period (discount) corporate bond selling at price p_t, and paying off principal ζ, where ζ is a normal random variable with mean 1 and constant variance σ^2. The uncertainy in return captures the risk of bankruptcy. The agent now maximizes expected lifetime utility

$$Max_{c_{0,t},c_{1,t+1}}E_t(1-\beta)\ln(c_{0,t})+\beta\ln(c_{1,t+1})$$

s.t.

$$\begin{aligned} c_{0,t} &= e_0 - p_t\gamma \\ c_{1,t+1} &= \gamma\zeta_t. \end{aligned}$$

Prove or disprove: The risk (i.e. variance) of the stochastic process generating ζ causes fewer bonds to be purchased relative to the certain-return case in 3(a).