

**Macroeconomics Qualifying Exam -- 303 Module**  
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**January, 2004**

Short questions.

Please answer all questions. Each part is worth 5 points.

1. Explain clearly why the capital stock is the state variable in a standard Cass-Koopmans optimal growth model. Use the market version of the model in your answer.
2. Discuss why technology shocks are now a controversial source of business cycle fluctuations.
3. Provide two reasons why rich countries have not reached a steady state in per capita income, and discuss why each reason is plausible from a modeling point of view.

Long questions.

Please answer either question 1 or question 2. Each part is worth 10 points.

1. Consider a standard two period life overlapping generations model with productive capital,  $K$ , and a social security system. Social security is pay-as-you-go where the young pay lump-sum tax  $\tau > 0$  and the old receive a transfer  $\sigma$ . The population grows geometrically,  $N_{t+1} = (1+n)N_t$ , where  $N_t$  is the number of youngsters at time  $t$ , and  $n \geq -1$ .

A consumer born at time  $t$  who has logarithmic utility solves the following expected utility maximization problem,

$$\begin{aligned} & \text{Max } c_{0,t}, c_{1,t+1} (1 - \beta) \ln(c_{0,t}) + \beta \ln(c_{1,t+1}) \\ & \text{s.t.} \\ & c_{0,t} = w_t - \tau - s_t \\ & c_{1,t+1} = R_{t+1} s_t + \sigma. \end{aligned}$$

- a) Find the present value budget constraint (PVBC) at time  $t$ .
- b) Find the government budget constraint at time  $t$  relating aggregate tax revenue and expenditures. Write the transfer  $\sigma$  in per youngster terms as a function of  $\tau$ .
- c) Use the (b) to write the PVBC with  $\tau$  the only fiscal policy parameter (i.e. substitute out  $\sigma$  using  $\tau$ ). Consider the model at a steady state, and derive a condition on parameters that, if true, causes lifetime resources to be higher with the social security system than without.
- d) In this society, social security reduces socio-political violence (SPV) that is causes output to be destroyed as poor oldsters riot. The production function in terms of per worker capital  $k_t \equiv K_t/N_t$  is  $k_t^\alpha \sigma^\eta$  for  $\alpha, \eta \in (0, 1)$ . Individuals and firms take the SPV effect  $\sigma^\eta$  as given when solving their optimization problems. Set up and solve the firm's profit maximization problem to find wages  $w_t$  and the interest factor  $R_{t+1}$  in terms of the state and parameters.

e) Find the PVBC where you substitute out  $w_t$  and  $R_{t+1}$  using the relations found in (d). Also substitute out  $\sigma$  in terms of  $\tau$  using the GBC. Now derive a condition showing that social security raises lifetime resources only if the capital stock is sufficiently large.

f) Find consumers' optimal savings function by solving their utility maximization problem above.

g) Construct the capital market clearing condition (CMCC) in per youngster terms at time  $t$  using  $\sigma$  instead of  $\tau$  throughout. Prove or disprove: for a given  $k_t$ ,  $k_{t+1}$  could be higher (i.e. under some conditions that you will specify) with the social security system than without.

h) [**Hard**] Characterize the dynamics of the CMCC. Do this by examining what happens when  $k_t$  approaches zero or infinity; differentiating the CMCC implicitly to show that  $dk_{t+1}/dk_t > 0$ .

Because of time constraints, do not attempt to find  $d^2k_{t+1}/dk_t^2 > 0$  but instead assume concavity.

i) Interpret this model. E.g. Suppose you are the advisor to the president of Bolivia and in 1 blue book page must explain to her in plain language the costs and benefits of instituting the social security system modeled here.

2. Consider a standard infinitely lived Cass-Koopmans model with productive capital. There is a single representative agent and population is constant and normalized to unity. Let the utility and the production functions have the standard properties.

a) Write down the planning version of the model when the agent is born at time  $t=0$ . Define all variables/functions you use

b) Find the FOC. Explain what it means in common words.

c) Now write down the market version of the model.

d) Find the FOC. Explain what it means in common words.

e) Prove that the FOC matches the ones above, explaining each step in your proof.

f) *Derive* the phase portrait for the market model version. Show all your work.

g) Now modify the planning version of the model to use the following production function:  $f(k) = Ak$ , where  $A \geq 1$  is a constant.

h) *Derive* the phase portrait. Show all your work.

i) Explain *how* and *why* the phase portrait in (h) differs from that in (f), with your answer being  $C^3$  (clear, concise and complete).