Macroeconomics Qualifying Exam – 303 Module
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Answer questions 1 and 2. Put **each answer** in a separate blue book with your assigned secret number on it and the question number.

1. Consider an overlapping generations model with productive capital where agents live two periods, working when young and retired when old, and differ within a generation by their productivity. Let i denote the "type" of an agent, with N^i the number of young type i agents and $\Sigma_{i=1}^{M} N^i \equiv N$ being the size of the young cohort, with $N^i = N^j$, $\forall i, j$. That is, there are M types of agents within every generation, where M is time invariant. When utility is logarithmic, agent i born at time t solves

$$(1-\beta)\ln(c_{0,t}^i) + \beta \ln(c_{1,t+1}^i)$$

s.t.

$$c_{0,t}^i = w_t^i - a_t^i$$
$$c_{1,t+1}^i = R_{t+1} s_t^i,$$

where w is wage, a is assets, $R = 1 + r - \delta$ is the returns on assets, with the depreciation rate on capital being $\delta \in [0,1]$ and r the interest rate.

a) Take the first order condition for a type i agent. What is the size of the state space (i.e. how many state variables are there)? Demonstrate the veracity of your claim for the size of the state space.

b) Let production be given by $Y = K^\alpha N^{1-\alpha}$, where Y is output and K is capital. Set up the firm's profit maximization problem and solve for the market interest factor R and the economy-wide wage w. There is an exogenous wage distribution M-dimensional vector ϵ with generic element ϵ^i such that $\Sigma_{i=1}^{M} \epsilon^i = 1$, and each agent is paid his or her marginal product. Then, the wage for a type i agent is $w^i = w\epsilon^i$. Further, let the elements in ϵ be ordered so that $\epsilon^1 < \epsilon^2 < ... < \epsilon^M$.

c) Combine the results of parts (a) and (b) to write the capital market clearing condition as a mapping from today's capital stock to tomorrow's capital stock. Use this mapping to produce a phase portrait of this economy's dynamics. Find all steady states analytically and determine the stability of each steady state by deriving arrows of motion which you will show in your phase portrait.

d) Now suppose that the wage distribution vector ϵ changes so ϵ^i, for $i = 1, 2, 3, 4$ are decreased by $0 < \gamma < 1$, and type M agents' wages are increased by $\eta = \gamma \Sigma_{i=1}^{4} \epsilon^i$, i.e. type M wages increase by the amount that types 1,2,3, and 4 decreased. Prove or disprove: this increase in inequality reduces the steady state capital stock.

e) Continuing with part (d), suppose that agents of type 1, 2, 3, and 4 can take time away from work and try to get their "stolen" income back from the type M agents. Without working out the details, describe in this situation how an increase in equality affects the economy. Specifically, what happens to the economy's growth path as the wage for agents of type 1,2, 3, and 4 continue to decrease, i.e. by 50 %, 60%, 70%, ... Compare your answer in this question to part (d) above. Limit your answer to 1 blue book page.

2. Consider a standard two period life overlapping generations model with lump-sum taxes paid by the young $\tau_t > 0$, for all times t. Taxes in this model are a random variable; that is, the tax rate τ_t at time t is **not** known for certain at time t, but must be forecasted. In particular, the government sets taxes so that they have an autoregressive form

$$\tau_t = \rho\tau_{t-1} + \epsilon_t$$

where ϵ_t is a normally distributed shock to the tax, with unconditional expected value of 0, and $\rho \in (0, 1)$. Assume for now that tax revenue is dumped into Lake Claremont each period. Lastly, the population grows geometrically, $N_{t+1} = (1 + n)N_t$, where N_t is the number of youngsters at time t, and $n \geq -1$.

A consumer born at time t solves the following expected utility maximization problem,

$$Max_{c_{0,t},c_{1,t+1}} EU(c_{0,t}, c_{1,t+1})$$

s.t.

$$\begin{aligned} c_{0,t} &= w_t - \tau_t - s_t \\ c_{0,t} &= R_{t+1}s_t. \end{aligned}$$

a) Take the first order condition for this model. What is/are the state variable(s) at time t?

b) Construct the capital market equilibrium condition in per youngster terms using only the state variable(s) and constants using the equilibrium wage from the firm's problem.

c) Draw a graph of the equilibrium dynamics in this model in k_t, k_{t+1} space for two cases in the same graph: (*i*) taxes are constant, $\tau_t = \tau > 0$ for all t; (*ii*) the tax is a random variable and follows the autoregressive expression given above. Include a 45 degree line and arrows of motion showing how the economy evolves. How many steady states are there for economy (ii)?

d) Describe what happens in your graph (ii) above by plotting k_t against time. What does the persistence of taxes mean for the persistence of variations in capital, output and consumption?

e) Is this a viable model of business cycles? By that, I mean do you think that this model could do a reasonably good job of replicating the data. Why or why not (answer in one blue book page or less)?

f) Compare this model of business cycles to the real business cycle model. In particular, tell me about this model's strengths and weaknesses compared to the RBC model, both from a technical point of view, and for realism. If the model in this question explained 90% of the variation in output, would it be a "better" business cycle model than the RBC model–why or why not? (answer in a max. of 1 blue book page).

g) Now suppose that taxes in this model are used for public investment, λ. The per worker government budget constraint is $\tau_t = \lambda_t$. In this case, the output per youngster is $y_t = k_t^\alpha \lambda_t^{1-\alpha}$. Write the per worker capital market equilibrium condition for this case using the government budget constraint to substitute out λ_t, i.e. the only variables should be k and τ. Prove or disprove: This model displays less persistence than the model in which tax revenue is unproductive.

h) Draw three phase portraits in the same graph for the model in part (f) when public investment amd taxes are nonstochastic, one in which taxes are very low; one in which taxes are moderate; and one with high taxes. In which case would output in the long-run be highest? Give me a brief explanation supporting this claim (answer in one blue book page or less).