Macroeconomics Qualifying Exam – 303 Module
Professor Paul J. Zak
Claremont Graduate University
May, 2001

Answer question 1 and *either* question 2 or 3. All parts are equally weighted.

1. Consider a two-period life overlapping generations model with a government sector. Agents work in the first period of life, earning wage w and save s for old age. In addition, all agents pay a proportional tax $\tau \in (0,1)$ on all types of income, i.e. on wages and on the interest income on savings. Leisure is not valued, and the old are retired and consume from their savings. Tax revenue has no direct effect on consumers' choices (i.e. government spending is unproductive). There is no uncertainty in this problem and population is constant and normalized to unity.

a) Write down the consumer's utility maximization problem defining all terms.

b) What is/are the state variable(s) for the problem above? Find the consumer's first order condition for consumption/savings?

c) Provide one or more conditions under which the following is true: an increase in taxes raises savings. Interpret this/these conditions.

d) Construct the capital market clearing condition. Specify the interest rate at which the capital market clears.

e) Under the condition(s) in (c), discuss (or informally prove) whether this model has a unique interior steady state (a formal proof is not required).

f) Write down the government budget constraint, when tax revenue is used to fund government consumption G, and the government issues one-period bonds B to cover any deficit. Bonds are reissued every period. For this model, define the government's primary budget deficit.

g) What is the equilibrium rate of return on bonds? Formally prove that bonds and capital have the same interest rate.

h) Now set $\tau_t = G_t = 0$, $\forall t$ and derive the phase portrait (including arrows of motion) for this economy. Identify each steady state and state its stability property.

i) Rederive the phase portrait and phase arrows for this model when $\tau_t = G_t = 0$, $\forall t$ and when there is a maximum amount of debt the government can issue, $B_t \leq \overline{B} < \infty$ $\forall t$. Assume that $\overline{B}$ is less than the value of debt at the interior steady state. Include a brief discussion of your approach to this problem.

2. Consider an optimal growth model with population growth. In this model, population grows stochastically, $N_{t+1} = (1+\gamma)N_t + \epsilon_{t+1}$ for $\epsilon \sim N(0, \sigma^2)$ and $\gamma > 0$. In this model, N_t is known at time t, but N_{t+1} is not known with certainty at t. Leisure is not valued.

a) Write down the planning version of the optimal growth model in per capita terms with population growth as specified above. Define all terms you use.

b) What is/are the state variable(s) for this model? What is/are the choice variables?

c) Find the Euler equation for this model.

d) State the complete set of optimality conditions for this model.

e) How would you evaluate this model as a model of business cycles? Compare it to the standard real business cycle model and tell me how well you expect the model in this section to perform. Please limit your analysis to 2 blue book pages.

3. Consider an optimal growth model with a constant population which is normalized to unity,

$$Max_c \Sigma_{t=0}^{\infty} \beta^t \ln(\frac{c_t}{\bar{c}}) \tag{1}$$

s.t.

$$\begin{aligned} c_t &= Ak_t - i_t \\ i_t &= k_{t+1} - (1-\delta)k_t, \end{aligned}$$

where $\bar{c} > 0$ is a reference level of consumption that agent compare themselves to (this is called *keeping up with the Jones'* utility).

a) Identify all prices in model (1). How many markets are there?

b) Find the Euler equation and demonstrate that this produces a linear second order difference equation in k.

c) State the complete set of optimality conditions for this model. Prove or disprove: An increase in $\bar{c}$ raises growth and increases the steady state level of capital.

d) Derive the arrows of motion for this model and construct a phase portrait.

e) How many steady states are there in this model? Fully characterize how this model's dynamics differ from the standard Cass-Koopmans model's dynamics.