Macroeconomics Qualifying Exam – 303 Module

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Answer 1 and **either** question 2 or 3. All questions are equally weighted.

1. Consider a two period life overlapping generations economy. Agents work in their first period of life, and are retired in the second. In this economy there is an infinite supply of an asset D (debt) which pays a time-invariant one-period return (principal and interest) set by the world market of $R^D = 1.05$. This economy also has private capital K which is used by firms in production. Other than the availability of debt D, this is a closed economy, so that the return on capital, call it R, is set by the domestic market. Debt has no effect on production, but simply serves as an asset that individuals can purchase with their savings if they so choose. Population is constant and is normalized to unity.

a) Using a general utility function that is continuous, strictly increasing, and concave, and satisfies the Inada conditions, write down the lifetime utility maximization problem for an agent born at time t. Please define any variables that you use.

b) What is/are the state variable(s) for this problem? What is/are the choice variable(s)?

c) Set up a representative firm's profit maximization problem using a general constant returns to scale production function that is continuous, strictly increasing, and concave, and satisfies the Inada conditions. Find the equilibrium wage at time t and return on capital from t to $t+1$.

d) Take the first order conditions for the utility maximization problem in (a) and solve for the optimal savings relation $s_t^\star$. Explicitly show the variables that savings depends on.

e) Construct the capital market clearing condition in terms of current and future state variable(s) and parameters.

f) Carefully and completely define a competitive equilibrium for this model.

g) State a theorem that guarantees that the equilibrium dynamics produce a unique interior steady state.

h) Under the assumption that the theorem you stated in part (g) holds, draw (you do not need to derive) the phase portrait for this economy.

i) Using your phase portrait as a guide, describe what happens to the dynamic evolution of the economy beginning from an initial capital stock $K_0 = .005$. In particular, tell me how the debt D impacts the dynamics (if at all).

j) Redraw the phase portrait when $R^D = 3.0$. Describe what happens in this case.

2. Consider a Cass-Koopmans model with physical capital. Population is constant and is normalized to unity. In this model the infinitely-lived representative agent values leisure. Time is normalized to unity.

a) For general utility and production functions that are both strictly increasing, continuous, and concave, and satisfy their respective Inada conditions, write down the planning version of the model. Be sure to define all variables that you use.

b) Identify the state and choice variables. Define a "planning problem" and a "competitive problem" in general.

c) Produce the optimality condition(s) for the planner. Identify the optimal variable (with a "$\star$") that each condition implicitly defines.

d) Carefully state the complete set of conditions required for an optimum to obtain for this problem.

e) Derive the dynamical behavior of this model by producing phase portraits. Please identify all steady states and *derive* arrows of motion.

f) Using the phase portrait (and equations of the model), characterize the impact of valued leisure on the dynamics of this model (i.e. vs. a model without valued leisure).

3. Consider a Solow model in which agents save $s \in (0,1)$ each period. Let population grow, $N_{t+1} = (1+n)N_t$, for $n > -1$.

a) Using a general constant returns to scale production function that is continuous, strictly increasing, and concave, and satisfies the Inada conditions, write down the aggregate capital market clearing condition for this model. Now write this in per capita terms.

b) How many steady states does this model have? Identify/find each one.

c) How much does the agent consume at time t?

d) Define the "golden rule." Now find the golden rule allocation for this model.

e) Prove or disprove: For any $s \in (0,1)$, the steady state per capita capital stock is greater than golden rule capital level.

f) Prove or disprove: This model has two and only two steady states.