Macroeconomics Qualifying Exam -- 303 Module
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Please answer question 1 and **either** question 2 **or** question 3. Each subpart is worth 10 points.

Q1. Consider a Cass-Koopmans model with an infinitely lived representative agent who obtains utility from consumption, c, and leisure, $\mathbf{l}$. The model is given by

$$Max_{ct,\, \mathbf{l}t} \sum_{t=0}^{\infty} b^t \left[U(c_{,t}) + V(\mathbf{l}_t)\right]$$

$$s.t.$$
$$c_t + i_t = f(k_t, h_t)(1-t)$$
$$k_{t+1} = (1-d)k_t + i_t$$
$$h_t + \mathbf{l}_t = 1,$$

where k is capital, h is labor hours, d Î $(0,1)$ is depreciation, b Î $(0,1)$ is the agent's discount factor, t Î $[0,1)$ is the tax rate, and i is investment. All tax revenue is dumped in the LA river. $U(\cdot)$, $V(\cdot)$ are neoclassical utility functions, and $f(\cdot)$ is a neoclassical production function: all have the standard properties and satisfy the Inada conditions. There is no population growth.

a) Identify the state and choice variables. How many markets are in this model?

b) Find the FOCs.

c) State the complete set of optimality conditions for the model.

d) *Derive* the phase portrait for the model. Identify all steady states and their stability properties. Show all your work.

e) Now let the tax rate t double, with $2t < 1$. i) Derive the mathematical effect of this change on the phase curves; ii) Show the effect of this change graphically by producing a phase portrait showing the dynamics for t and $2t$.

f) Now suppose that the tax $t = .95$. i) Redraw the phase portrait and ii) clearly, carefully, and concisely (C^3) discuss what this tax rate does to the economy's dynamics.

g) Prove or disprove: The tax rate distorts the planner's labor supply decision.

h) Now suppose that taxes pay for a lump-sum transfer, s, to the representative agent. The new resource constraint is: $c_t + i_t = f(k_t, h_t)(1-t) + s$, and otherwise the model is unchanged. Prove or disprove: For this model, the tax *t* distorts the planner's labor supply decision.

Q2. Consider a pure exchange two period life model with a fixed number of shares of stock that can be used to transfer resources intertemporally and is traded in a stock market. Population is constant and normalized to 1. A consumer who has logarithmic utility solves the following utility maximization problem,

$$\begin{aligned} &Max_{c,\,c¢}\ ln(c) + b\ ln(c¢) \\ &s.t. \\ &c = e - g\,p \\ &c¢ = (p¢ + d¢)\,g \end{aligned}$$

where variables without a prime (¢) are in the current period, and those with a prime are in the following period. In this model, $b\ Î\ (0,1)$ is the agent's discount factor, *c* is consumption, $e>0$ is endowment of the consumption good, *p* is the stock price, *g* is shares of stock (assumed to be a continuous variable), and *d* is dividends paid per share of stock. Next period's price and dividend are nonstochastic.

a) Find the optimal demand for shares of stock by the agent, g^*.

b) Prove or disprove: i) shares demanded by the agent fall as current price *p* rises, and ii) shares demanded rise as future price $p¢$ rises.

c) Find the demand for stocks when price $p=0$. Is this finding the result of a market failure? (Yes or no, and explain using standard economic theory).

d) Find the stock market clearing price, p^*. Explain what this finding means C^3.

e) We now modify the model so that agents pay a proportional transaction cost $d\ Î\ (0,1)$ to a broker (who is not part of the model) each time she buys or sells a share of stock. This model is:

$$\begin{aligned} &Max_{c,\,c¢}\ ln(c) + b\ ln(c¢) \\ &s.t. \\ &c = e - g\,(1+d)p \\ &c¢ = (p¢ + d¢)\,g\,(1+d) \end{aligned}$$

Show mathematically how *d* affects i) the agent's demand for stocks g^*, and ii) the number of stocks held in equilibrium.

f) Suppose that next period's price $p¢$ is stochastic, $p¢ \sim N(m,s)$. i) Write down the complete model for this case and derive the optimal demand for shares of stock by the agent, g^*, and ii) Prove of disprove: shares purchased rise as the expected future price $E(p¢) = m$ rises.

Q3. Consider a pure exchange two period life overlapping generations model with a proportional endowment tax t Î $(0,1)$, and a consumption tax $s>0$. All tax revenue is sent outside the economy. The population is constant and normalized to 1. A consumer solves the following utility maximization problem,

$$
\begin{aligned}
&Max_{\ c0,\ c1}\ U(c_0, c_1) \\
&s.t. \\
&c_0 \quad = e_0(1-t)- s \\
&(1+s)c_1 = e_1 + R\ s.
\end{aligned}
$$

The endowments are $e_0,e_1 >> 0$, c_0,c_1 are first and second period consumption, R is the yield on savings s, and $U(\cdot)$ is a standard neoclassical utility function satisfying the Inada conditions.

a) Find the FOC. Identify the factors upon which the optimal savings relation s^* depends.

b) i) Identify (don't derive) the signs of ds^*/dt and ds^*/ds, and ii) explain how the agent's behaviors cause these results.

c) Let $U(c_0, c_1) = (1-b)\ ln(c_0) + b\ ln(c_1)$. Find the optimal savings relation s^*.

Complete the next three questions using the log utility function.

d) Identify each market in this model, including supply, demand, and the price. Define a general equilibrium for this model C[3].

e) Find the equilibrium interest factor R*. Derive and sign dR^*/dt and dR^*/ds.

f) Now modify the model by removing the consumption tax and taxing the endowment in both periods. The model is

$$
\begin{aligned}
&Max_{\ c0,\ c1}\ (1-b)\ ln(c_0) + b\ ln(c_1) \\
&s.t. \\
&c_0 = e_0(1-t)- s \\
&c_1= e_1(1-t) + R\ s.
\end{aligned}
$$

Prove or disprove: The equilibrium interest factor for this model is higher than for the model with the consumption tax when $s=t$.