Macroeconomics Qualifying Exam -- 303 Module
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May, 2005

Please answer ***2*** *of the following 3 questions. Each subpart is worth 10 points.*

1. Consider a Solow model with population growth. Let the production function be $F(K) = AK^3$, where K is the aggregate capital stock and $A>0$.

a) Write down the *aggregate* equilibrium dynamical system, defining all terms that you use.

b) Using (a), *derive* the equilibrium dynamic system in per capita terms.

c) Using (b) *derive* the equilibrium dynamics, including: finding all steady states; drawing a phase portrait; and showing arrows of motion.

d) *Prove* the stability properties of all steady states by examining the slope of the dynamical system at the steady state.

e) Discuss how the dynamical system you have characterized does, or does not, help explain cross-country data on economic growth in the world. Limit your answer to one blue book page or less, and be specific about what the model predicts.

2. Consider a two period life pure exchange overlapping generations model. The population is constant and is normalized to unity. Consumers in this world solve

$$Max_{c0,\, c1}\ \frac{c_0^{1/2}}{2} + \frac{\alpha c_1^{1/2}}{2}$$

s.t.

$c_0 = e_0 - s$

$c_1 = e_1 + Rs,$

for $\alpha, e_0, e_1 > 0$. All variables have the standard definitions.

a) Take the first order condition and solve for the optimal savings relation s^*.

b) Prove or disprove: First period consumption is a normal good.

c) Define a competitive equilibrium for this model fully and completely.

d) Now we introduce a fixed stock of money, $m>0$, into this model. Define a competitive equilibrium for this variant of the model. Solve for the equilibrium interest factor, call it R^m.

e) Find the equilibrium interest factor for the model without money and call it R^*. Without doing the mathematics, argue convincingly using economic theory that that $R^* < R^m$.

3. Consider a standard infinitely lived Cass-Koopmans model with productive capital. There is a single representative agent and population is constant and normalized to unity. Let the utility and the production functions have the standard properties.

a) Write down the planning version of the model when the agent is born at time t=T. Define all variables/functions that you use

b) Find the FOC(s). State the full set of optimality conditions for this model.

c) *Derive* the phase portrait for the model. Find all steady states, note their stability properties, and show your work.

d) Now modify the model to use the following production function: $f(k) = Ak^2$, where A > 0 is a constant. *Derive* the phase portrait. Find all steady states, note their stability properties, and show your work.

e) Identify the differences between the dynamics of the two versions of the model with your answer being C^3 (clear, concise and complete). Which model do you think better explains cross-country growth rate data and why?