Macroeconomics Qualifying Exam – 303 Module
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Answer question 1 and either question 2 or 3. Put **each answer** in a separate blue book with your assigned secret number on it and the question number.

1. Consider an overlapping generations model with productive capital where agents live two periods, working when young and retired when old, and differ within a generation by their productivity. Let i denote the "type" of an agent, with N^i the number of young type i agents and $\Sigma_{i=1}^{M} N^i \equiv N$ being the size of the young cohort. That is, there are M types of agents within every generation, where M is time invariant. When utility is logarithmic, agent i born at time t solves

$$(1-\beta)\ln(c^i_{0,t}) + \beta \ln(c^i_{1,t+1})$$

s.t.

$$c^i_{0,t} = w^i_t - a^i_t$$
$$c^i_{1,t+1} = R_{t+1} s^i_t,$$

where w is wage, a is assets, $R = 1 + r - \delta$ is the returns on assets, with the depreciation rate on capital being $\delta \in [0,1]$ and r the interest rate.

a) Take the first order condition for a type i agent. What is the size of the state space (i.e. how many state variables are there)? Demonstrate the veracity of your claim for the size of the state space.

b) Let production be given by $Y = K^{\alpha}\Sigma_{i=1}^{M}(N^i)^{1-\alpha}$, where Y is output and K is capital. Set up the firm's profit maximization problem and solve for the market interest factor R and the wage w^i for each type of agent i. Explain the economic intuition for why the interest factor R does not vary across agents, but wages do.

c) Combine the results of parts (a) and (b) to write the capital market clearing condition as a mapping from today's capital stock to tomorrow's capital stock. Use this mapping to produce a phase portrait of this economy's dynamics. Find all steady states analytically and determine the stability of each steady state by deriving arrows of motion which you will show in your phase portrait.

d) Now suppose that the economy is not at a steady state and there is a one-time unexpected in-migration of type $i = b$ agents, so that N^b is permanently raised $(1 \le b \le M)$. What is the effect of this migration on the type b agents who already live in this country? What is the effect of migration on the aggregate dynamics of this economy? Show this change by drawing a new phase portrait.

e) Characterize the evolution of the distribution of income as this economy grows. Does your answer depend on the choice of the utility function?

2. Consider an overlapping generations model with productive capital where agents live three periods and choose how many children to have. Agents within a generation are heterogeneous in their wage. In the first period, agents are children and their consumption is funded by their parents. Children do not receive utility from consuming (since they don't choose their consumption). In the second period of life (age 0), agents supply labor inelastically, consume and save. In the third period (age 1), agents are retired and consume from their savings. Use the following notion: c_i is consumption at age $i = 0, 1$, t is time, which is discrete, w^i is wage, a^i is assets, $R = 1+r-\delta$ is the net return on savings, where $\delta \in [0,1]$ is the depreciation rate on capital and r is the interest rate. Parents have all their children, b^i, when they are age 0, receive utility from children only at that time, and pay a cost of e^i per child to support them during the first period of their children's lives. (We ignore marriage here–agents reproduce asexually–and disregard the integer constraint on the number of children). Assume that the following functional forms hold for a type i agent,

$$U(c_0^i, c_1^i, b^i) = (1-\beta)\ln(c_0^i) + \beta\ln(c_1^i) + \gamma\ln(b^i),$$

and

$$f(k) = k^\alpha$$

for $\alpha, \beta \in (0,1)$, $\gamma > 0$ and $k \equiv \frac{K}{N}$ per capita capital with K aggregate capital and N population. Initial capital given and is strictly positive, $k_0 > 0$.

a) Write down the consumer i's lifetime utility maximization problem when she is born at time $t-1$ from the information in this question using the utility function above and supplying the proper budget constraint.

b) What are the state and choice variable(s) for agent i at time t? Derive the consumer's optimality condition(s). Let w be the average wage and ϵ by a N-dimensional vector with $\Sigma_{i=1}^N \epsilon^i = 1$, with the wage of a type i agent being $w^i = \epsilon^i w$. Find the wage of a type i agent in terms of per capita capital and the scalar ϵ^i. Also find the interest factor R.

c) Using the first order condition for the number of children to have, let the cost of children by proportional to one's wage, $e^i = (w^i)^\eta$, with $\eta < 1$ for $\eta < w^\star$ and $\eta > 1$ for $\eta \geq w^\star$, where $w^\star$ is some value of the average wage that is less than the steady state average wage. For a low value of initial capital, $k_0 \approx 0$, describe the dynamics of fertility as this economy grows. Assume that for *any* value of the wage, that each agent has at least one child (i.e. minimum fertility is 1). Include a diagram of b^i as a function of k_t. Discuss the *distribution* of fertility across agents, and aggregate fertility $B_t = \Sigma_{i=1}^N b_t^i$ over time.

d) Draw the phase portrait in k_t, k_{t+1} space for a given vector ϵ and determine the dynamics of this economy. Prove or disprove: If preferences for children are sufficiently high, the economy will not grow at a positive rate.

e) Now suppose that average wages are given by $w = Ak$, for $A > 0$. Draw a phase-portrait of the dynamics of the system. Prove that as $k \rightarrow \infty$ all types of agents will have only one child. Is this case empirically relevant?

3. Consider an overlapping generations model with productive capital where agents live two periods and there is government debt and a primary budget deficit. Let population growth be geometric, $N_{t+1} = (1+n)N_t$, with $N_0 = 1$. The consumer's problem at time t is given by

$$Max_{c_0,c_1} U(c_{0,t}, c_{1,t+1})$$

s.t.

$$\begin{aligned} c_{0,t} &= w_t - s_t - \tau \\ c_{1,t+1} &= R_{t+1} s_t. \end{aligned}$$

The government budget constraint at time t is

$$N_t \tau + B_{t+1} = R_t B_t + G.$$

The primary budget deficit is $Q_t = G - N_t \tau > 0$, where G is government consumption which is time invariant.

a) Find the agent's optimality conditions and define a competitive equilibrium.

b) What is the asset market clearing condition in per youngster terms? Use lower case letters to denote per youngster amounts, i.e. $b \equiv \frac{B}{N}$, etc.

c) Draw a phase portrait of this economy in k_t, b_t space. Find all steady states and derive arrows of motion.

d) Examine the effect of the primary budget deficit on the dynamics of this economy by considering what happens when initial capital k_0 is very small.

e) Examine the effect of the primary budget deficit on the dynamics of this economy by considering the case when per youngster government spending is proportional to per youngster capital, i.e. $g = \phi k$, for some $\phi > 0$.