Macroeconomics Qualifying Exam – 303 Module
Professor Paul J. Zak
Claremont Graduate University
October, 1997

Answer both questions. All parts are equally weighted.

1. Consider an overlapping generations model with productive where agents live two periods. In the first period, agents supply labor inelastically, consume and save. In the second period, agents are retired and consume from their savings. Use the following notion: c_i is consumption at age $i = 0, 1$, t is time, which is discrete, w is wage, s is savings, $R = 1 + r - \delta$ is the net return on savings, where $\delta \in [0, 1]$ is the depreciation rate on capital. Assume that population is constant and is normalized to unity and that the following functional forms hold,

$$U(c_0, c_1) = (1 - \beta)\ln(c_0) + \beta \ln(c_1),$$

and

$$f(k) = A\ln(1 + k)$$

for $A > 0$ and $\beta \in (0, 1)$. Initial capital given and is strictly positive, $k_0 > 0$.

a) What are the state and choice variable(s)? Derive the consumer and firm optimization conditions.

b) Write down the asset market clearing condition. Define a competitive equilibrium and show that all the conditions of this definition are satisfied for the above model.

c) Draw the phase portrait and determine the dynamics of the system. How many stationary equilibria are there in this model? Prove or disprove: The value of A affects the number of steady states.

d) Now suppose that wages are given by $w = Bk$, for $B > 0$. Show that the dynamics of the system change when B is below or above some threshold value. Demonstrate your claim by drawing phase portraits. Interpret the parameter B and argue the each of the dynamics you've depicted could describe the data from a particular "type" of country (i.e. map the dynamics of the model into stylized data).

2. Consider an overlapping generations model with productive capital where agents live two periods. Use the same notation as in question #1, and assume that agents supply labor inelastically in the first period of their lives and are retired in the second period. Suppose further that the production technology is given by

$$F(K_t, N) = K_t^{\alpha} N^{1-\alpha}$$

for $\alpha \in (0, \frac{1}{2})$, and population, N, is assumed constant. Let preferences be represented by the utility function

$$(1 - \beta)\ln(c_{0,t}) + \beta \ln(c_{1,t+1}).$$

a) What is the asset market clearing condition in per capita terms? Define the "golden rule" allocation.

b) Derive the golden rule level of capital. Find a restriction on α that guarantees that steady state capital is less than the golden rule level of capital.

c) Now, suppose that there is no capital in this economy. Assume that agents consume from an endowment $\{e_0, e_1\} >> 0$. Use the same form for utility as that given above. Find the market clearing interest factor, R^*. Be sure to state the equilibrium condition you are using clearly. Explain in one or two sentences why you have identified the correct equilibrium condition.

d) Next, introduce pieces of paper with my picture on them into the economy of part (c). Call these pieces of paper money and assume that there is a fixed supply of them, say M, which are uniformly given to the initial old generation. What is the market clearing condition when the endowment is again $\{e_0, e_1\} >> 0$? Find the market clearing interest factor and derive a restriction on e_0 than guarantees that R is strictly positive.