

Macroeconomics Qualifying Exam -- 303 Module
Professor Paul J. Zak
Claremont Graduate University
September, 2005

This qual has a single question to answer. Note, though, that the sub-questions beginning with part (i) can be considered a separate question. Each sub-part is worth 10 points.

Q1. Consider a Cass-Koopmans model with an infinitely lived representative agent who obtains utility from consumption, c , and pays a tax, $\tau \in [0, 1)$. The model is given by

$$\begin{aligned} & \text{Max}_{c,t} \sum_{t=0}^{\infty} \beta^t U(c_t) \\ & \text{s.t.} \\ & c_t + i_t = w_t (1-\tau) + r_t k_t \\ & k_{t+1} = (1-\delta)k_t + i_t \end{aligned}$$

where k is capital, $\delta \in [0, 1]$ is depreciation, i is investment, and $\beta \in (0, 1)$ is the agent's discount factor. Tax revenue is dumped in the San Pedro harbor (i.e. tax revenue is unproductive). $U(c)$ is a neoclassical utility function, and there is a neoclassical production function $f(k)$. Both $U(\bullet)$ and $f(\bullet)$ have the standard properties and satisfy the Inada conditions. There is no population growth and you may consider population normalized to unity.

- a) Is this a competitive or planning model? Support your answer.
- b) Prove or disprove: the equilibrium allocation from the model above is equivalent to a competitive/planning model with similar constraints. Be sure to write down the "other" type of model your proof is based upon.
- c) State the government budget constraint. Prove or disprove: when $\tau > 0$ the planner optimally directs the agent to work less time than when $\tau = 0$.
- d) Find the FOCs of the model.
- e) State the complete set of optimality conditions for the model.
- f) Identify all steady states and their stability properties. Derive the arrows in the phase portrait showing when the dynamics lead to growth or contraction. Show all your work, but do not draw the phase portrait yet.
- g) Prove or disprove analytically: $\tau > 0$ affects the model's dynamics.
- h) Depict your result in part (g) by drawing the phase portrait for $\tau = 0$ and $\tau > 0$.

Macroeconomics Qualifying Exam -- 303 Module
Professor Paul J. Zak
Claremont Graduate University
September, 2005

i) Now suppose the tax revenue is used to purchase a public good (e.g. parks), σ , over which agents have preferences. This new model is

$$\text{Max}_{c_t, \sigma_t} \sum_{t=0}^{\infty} \beta^t [U(c_t) + V(\sigma_t)]$$

s.t.

$$c_t + i_t = w_t (1 - \tau) + r_t k_t$$

$$k_{t+1} = (1 - \delta)k_t + i_t$$

$$\sigma_t = w_t \tau,$$

where $V(\bullet)$ is continuous, strictly increasing, and concave with $V(0)=0$. Find the FOCs for this model.

j) Using words, briefly describe the FOC for σ , i.e. how the agent determines the optimal tradeoff between how much taxes she wants to pay to obtain the public good.

k) Prove or disprove: The public good is a normal good.

l) Now let $U(c) = \ln(c)$, $V(\sigma) = \gamma \ln(\sigma)$, and $f(k) = k^\alpha$ for $\gamma > 0$, $\alpha \in (0, 1)$. Find all steady states.

m) Derive the arrows of motion and draw the phase portrait.

n) Prove or disprove: The preference for public goods fundamentally changes the dynamics of the model relative to the standard Cass-Koopmans model.

o) Suppose you are a libertarian and do not think the government should be building parks, $V(\sigma) = 0$, for all $\sigma \geq 0$. Prove that: the optimal tax rate in this case is $\tau = 0$.

p) Prove or disprove using this model: the size of government will continually rise (known as "Wagner's rule").