Macroeconomics Qualifying Exam – Part 1
Claremont Graduate School
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Answer 2 of the 3 questions. All parts are equally weighted.

1. Consider the Lucas "fruit-tree" asset pricing model in which individuals are infinitely lived. Let γ_t be the number of shares purchased of a given security at time t at price p_t. Dividends on shares are nonstochastic and are denoted by d_t and agents receive an endowment of consumable "fruit", denoted by e_t.

a) Let $\beta \in (0,1)$ be the agent's subjective discount factor. Set up this model and derive the fundamental asset pricing equation when prices are nonstochastic.

b) Let

$$R_{t+1} \equiv \frac{p_{t+1} + d_{t+1}}{p_t}$$

and compare this optimality condition to an infinitely lived agent endowment economy in which instead of a stock market, agents make loans among themselves. What is/are the equilibrium condition(s) in each variant of the model?

c) What is the return on a fruit-tree when agents are perfect consumption smoothers? Now suppose that the current asset price is known, but future asset prices are stochastic. Set up this version of the model and derive the fundamental asset pricing equation. Interpret the investment decision as an insurance mechanism.

d) Discuss the relationship between the version of the model in part (c) and the Capital Asset Pricing Model (CAPM). [Hint: *Don't* derive the CAPM.]

2. Consider a neoclassical growth model in which agents live two periods and are partitioned into H household types, with $h \in \{1, 2, ...H\}$ denoting the type. Each household type earns a different wage, ω_h when young, is retired when old and does not value leisure. Assume that all individuals are born without assets and that population is constant. Denote the number of type h individuals as N_h with total population given by

$$N = \Sigma_{h=1}^{H} N_h.$$

a) Write down a generic lifetime utility maximization problem for an agent living in such an economy. Define all variables and parameters that you use. Derive the optimality condition for consumers.

b) Define a competitive equilibrium. Under what conditions would such an equilibrium exist?

c) Let

$$U(c_h) = \ln(c_h)$$

and

$$F(K_t, N) = K_t^{\alpha}(\Sigma_{h=1}^{H} N_h)^{1-\alpha}$$

for $\alpha \in (0,1)$. Explicitly derive the asset market clearing condition and draw a phase portrait of this economy.

d) Repeat part (c) assuming that there are only 2 household types where type 1 is subject to a lump-sum tax τ as youngsters which is transferred to type 2 agents lump-sum when they are youngsters. Let σ denote the transfer. How does this change aggregate behavior, if at all? Demonstrate using a phase portrait.

3. Consider an overlapping generations model with productive capital and government debt where agents live two periods and solve

$$Max_{c_{0,t}, c_{1,t+1}} U(c_{0,t}, c_{1,t+1})$$

s.t.

$$\begin{aligned} c_{0,t} &= w_t - s_t \\ c_{1,t+1} &= R_{t+1} s_t \\ N_{t+1} &= (1+n) N_t, \end{aligned}$$

where c_i is consumption at age $i = 0, 1$, t is time, w is wage, N is population with $n > -1$, s is savings, $R = 1 + r - \delta$ is the net return on savings, where $\delta \in [0,1]$ is the depreciation rate on capital. Assume that the following functional forms hold,

$$U(c_0, c_1) = (1-\beta)\ln c_0 + \beta \ln c_1,$$

and

$$F(K, N) = K^{\alpha} N^{1-\alpha}$$

for $\alpha, \beta \in (0,1)$. In addition, there is a government that functions solely to roll over its debt in perpetuity using the budget constraint

$$B_{t+1} = R_t B_t$$

where B is the aggregate stock of government debt which has the same return as physical capital.

a) Derive consumer and firm optimization conditions. What is the asset market clearing condition in per capita terms? Interpret this condition. What is/are the state variable(s)?

b) How many stationary equilibria are there in this model? Identify each one. Draw the phase portrait and determine the dynamics of the system.

c) Discuss the stability of each steady state equilibrium. Using a phase diagram, determine the dynamic path of this economy when government debt is zero. Based on this analysis, what is the effect on economic growth of (nonzero) government debt?