Claremont Graduate University
A MEMBER OF THE CLAREMONT COLLEGES

Institute of Mathematical Sciences

GRADUATE MATH COURSES

The following list is indicative of our course offerings. Not all courses are offered every year. Unless otherwise indicated, all of these are four-unit courses.

MATH 231
Principles of Real Analysis I. Countable sets, least upper bounds, and metric space topology including compactness, completeness, connectivity, and uniform convergence. Related topics as time permits.

MATH 232
Principles of Real Analysis II. A rigorous study of calculus in Euclidean Spaces including multiple Riemann Integrals, derivatives of transformations, and the inverse function theorem. Prerequisite: Math 231.

MATH 235
Complex Analysis. Topics include but not limited to: Algebraic properties of complex numbers. Topological properties of the complex plane. Differentiation and holomorphic functions. Complex series, power series. Local and global Cauchy's theorem. Mobius transformation. Cauchy's integral theorem. Classification of singularities, Calculus of residues, contour integration. Conformal Mapping. If time permits, the Laplace and the Fourier transforms. Prerequisite: linear algebra; Math 231 recommended.

MATH 236
Complex Variables and Applications. Complex differentiation, Cauchy-Riemann equations, Cauchy integral formula, Taylor and Laurent expansions, residue theory, contour integration including branch point contours, uses of Jordan's lemma, Fourier and Laplace transform integrals, conformal mapping.

MATH 239
Fourier Analysis. Fourier analysis begins with the examination of the difficulties involved in attempting to reconstruct arbitrary functions as infinite combinations of elementary trigonometric functions. Topics in this course will include Fourier series, summability, types and questions of convergence, and the Fourier transform (with, if time permits, applications to PDEs, medical imaging, linguistics, and number theory).

MATH 240
Modern Geometry. Geometry from a modern viewpoint. Euclidean geometry, discrete geometry, hyperbolic geometry, elliptical geometry, projective geometry, and fractal geometry. Additional topics may include algebraic varieties, differential forms, or Lie groups.

MATH 241
Hyperbolic Geometry. An introduction to hyperbolic geometry in dimensions two and three. Topics will include: Poincaré disk model, upper half-space model, hyperbolic isometries, linear fractional transformations, hyperbolic trigonometry, cross-ratio, hyperbolic manifolds, and hyperbolic knots.

MATH 242
Differential Geometry. Curves and surfaces, Gaussian curvature, isometries, tensor analysis, covariant differentiation with applications to physics and geometry (intended for physicists and mathematicians). Prerequisite: Math 231 recommended.

MATH 243
Topics in Geometry. Selected topics in Riemannian geometry, low dimensional manifold theory, elementary Lie groups and Lie algebra, and contemporary applications in mathematics and physics. Prerequisite: permission of instructor.

MATH 244
Algebraic Topology. An introduction to algebraic topology. Basics of category theory, simplicial homology and cohomology, relative homology, exact sequences, Poincare duality, CW complexes, DeRahm cohomology, applications to knot theory.

MATH 245
Topics in Geometry and Topology. Topic varies from year to year and will be chosen from: Differential Topology, Euclidean and Non-Euclidean Geometries, Knot Theory, Algebraic Topology, and Projective Geometry.

MATH 247
Topology. Topological spaces, product spaces, quotient spaces, Hausdorff spaces, compactness, connectedness, path connectedness, fundamental groups, homotopy of maps, and covering spaces. Corequisite: Math 231 or permission of instructor.

MATH 248
Knot Theory. An introduction to the theory of knots and links from combinatorial, algebraic, and geometric perspectives. Topics will include knot diagrams,p-colorings, Alexander, Jones, and HOMFLY polynomials, Seifert surfaces, genus, Seifert matrices, the fundamental group, representations of knot groups, covering spaces, surgery on knots, and important families of knots. Prerequisite: Topology (Math 247), or Algebra (Math 271), or permission of instructor.

MATH 249
Discrete Geometry. The goal of this course is to introduce students to the basics of discrete and convex geometry. Topics covered will include convex bodies, lattices, quadratic forms, and interactions between them, such as the fundamentals of Minkowski's theory, shortest vector problem, reduction algorithms, LLL, and connections to computational complexity and theoretical computer science. Additional topics may include an introduction to optimization questions, such as packing, and covering problems.

MATH 250

Statistical Methods for Clinical Trials Data. A second course in biostatistics. Emphasis on the most commonly used statistical methods in pharmaceutical and other medical research. Topics such as design of clinical trials, power and sample size determination, contingency table analysis, odds ratio and relative risk, survival analysis.

MATH 251

Probability. The main elements of probability theory at an intermediate level. Topics include combinatorial analysis, conditional probabilities, discrete and continuous random variables, probability distributions, central limit theorem, and numerous applications. Students may not receive credit for both Math 251 and Math 257.

MATH 252

Statistical Theory. This course will cover in depth the mathematics behind most of the frequently used statistical tools such as point and interval estimation, hypothesis testing, goodness of fit, ANOVA, linear regression. This is a theoretical course, but we will also be using R statistical package to gain some hands on experience with data.

MATH 253

Bayesian Statistics. An introduction to principles of data analysis and advanced statistical modeling using Bayesian inference. Topics include a combination of Bayesian principles and advanced methods; general, conjugate and noninformative priors, posteriors, credible intervals, Markov Chain Monte Carlo methods, and hierarchical models. The emphasis throughout is on the application of Bayesian thinking to problems in data analysis. Statistical software will be used as a tool to implement many of the techniques.

MATH 254

Computational Statistics. An introduction to computationally intensive statistical techniques. Topics may include: random variable generation, Markov Chain Monte Carlo, tree based methods (CART, random forests), kernel based techniques (support vector machines), optimization, other classification, clustering & network analysis, the bootstrap, dimension reduction techniques, LASSO and the analysis of large data sets. Theory and applications are both highlighted. Algorithms will be implemented using statistical software.

MATH 255

Time Series. An introduction to the theory of statistical time series. Topics include decomposition of time series, seasonal models, forecasting models including causal models, trend models, and smoothing models, autoregressive (AR), moving average (MA), and integrated (ARIMA) forecasting models. Time permitting we will also discuss state space models, which include Markov processes and hidden Markov processes, and derive the famous Kalman filter, which is a recursive algorithm to compute predictions. Statistical software will be used as a tool to aid calculations required for many of the techniques.

MATH 256

Stochastic Processes. Continuation of Math 251. Properties of independent and dependent random variables, conditional expectation. Topics chosen from Markov processes, second order processes, stationary processes, ergodic theory, Martingales, and renewal theory. Prerequisite: Math 251 or permission of instructor.

MATH 257

Intermediate Probability. 2-unit half-semester course covering approximately the second half of Math 251. Continuous random variables; distribution functions; joint density functions; marginal and conditional distributions; functions of random variables; conditional expectation; covariance and correlation; moment generating functions; law of large numbers; Chebyshev's theorem and central limit theorem. Students may not receive credit for both Math 251 and Math 257.

MATH 258

Statistical Linear Models. An introduction to analysis of variance (including one-way and two-way fixed effects ANOVA) and linear regression (including simple linear regression, multiple regression, variable selection, stepwise regression and analysis of residual plots). Emphasis will be on both methods and applications to data. Statistical software will be used to analyze data. Prerequisite: Math 252 or permission of instructor.

MATH 259

Methods of Applied Probability and Statistics (FORMERLY MATH 262). Covers approximately the second half of Math 251 and the second half of Math 252. Probability, Random Variables and their Distributions, Special Probability Distributions, Joint Distributions, Properties of Random Variables, Functions of Random Variables, Limiting Distributions, Statistics and Sampling Distributions, Point Estimation, Sufficiency and Completeness, Interval Estimation, Tests of Hypothesis. The pace will be rapid, covering roughly a chapter per week. Prerequisite: an undergraduate semester course in probability and statistics.

MATH 260

Monte Carlo Methods. This course introduces concepts and statistical techniques that are critical to constructing and analyzing effective simulations, and discusses certain applications for simulation and Monte Carlo methods. Topics include random number generation, simulation-based optimization, model building, bias-variance trade-off, input selection using experimental design, Markov chain Monte Carlo (MCMC), and numerical integration. Prerequisite: Math 251.

MATH 262

Machine Learning. Machine Learning (ML) is the process of discovering patterns in large data sets using techniques from mathematics, computer science and statistics. Applications range from biology and neuroscience to history, linguistics, and economics. In this course students will learn the mathematics and implementation of classical ML algorithms such as regression, K-Means, and kNN. Students will also be introduced to neural networks and their implementation in Python.

MATH 264

Scientific Computing. Computational techniques applied to problems in the sciences and engineering. Modeling of physical problems, computer implementation, analysis of results; use of mathematical software; numerical methods chosen from: solutions of linear and nonlinear algebraic

equations, solutions of ordinary and partial differential equations, finite elements, linear programming, optimization algorithms, and fast-Fourier transforms.

MATH 265

Numerical Analysis. An introduction to the theory and methods for numerical solution of mathematical problems. Core topics include: analysis of error and efficiency of methods; solutions of linear systems by Gaussian elimination and iterative methods; calculation of eigenvalue and eigenvectors; interpolation and approximation; numerical integration; solution of ordinary differential equations.

MATH 267

Complexity Theory. Specific topics include finite automata, pushdown automata, Turing machines, and their corresponding languages and grammars; undecidability; and complexity classes, reductions, and hierarchies.

MATH 268

Algorithms. Algorithm design, computer implementation, and analysis of efficiency. Discrete structures, sorting and searching, time and space complexity, and topics selected from algorithms for arithmetic circuits, sorting networks, parallel algorithms, computational geometry, parsing, and pattern-matching.

MATH 269

Representations of High Dimensional Data. In today's world, data is exploding at a faster rate than computer architectures can handle. For that reason, mathematical techniques to analyze large-scale objects must be developed. One mathematical method that has gained a lot of recent attention is the use of sparsity. Sparsity captures the idea that high dimensional signals often contain a very small amount of intrinsic information. In this course, we will explore various mathematical notions used in high dimensional signal processing including wavelet theory, Fourier analysis, compressed sensing, optimization problems, and randomized linear algebra. Students will learn the mathematical theory, and perform lab activities working with these techniques.

MATH 271

Abstract Algebra I. Groups, rings, fields and additional topics. Topics in group theory include groups, subgroups, quotient groups, Lagrange's theorem, symmetry groups, and the isomorphism theorems. Topics in Ring theory include Euclidean domains, PIDs, UFDs, fields, polynomial rings, ideal theory, and the isomorphism theorems. In recent years, additional topics have included the Sylow theorems, group actions, modules, representations, and introductory category theory.

MATH 272

Abstract Algebra II: Galois Theory. Topics covered will include polynomial rings, field extensions, classical constructions, splitting fields, algebraic closure, separability, Fundamental Theorem of Galois Theory, Galois groups of polynomials and solvability. Prerequisite: Math 271. This course is independent of Math 274 (Abstract Algebra II: Representation Theory), and students may receive credit for both courses.

MATH 273

Linear Algebra. Topics may include approximation in inner product spaces, similarity, the spectral theorem, Jordan canonical form, the Cayley Hamilton Theorem, polar and singular value decomposition, Markov processes, behavior of systems of equations.

MATH 274

Abstract Algebra II: Representation Theory. Topics covered will include group rings, characters, orthogonality relations, induced representations, application of representation theory, and other select topics from module theory. Prerequisite: Math 271. This course is independent of Math 272 (Abstract Algebra II: Galois Theory), and students may receive credit for both courses.

MATH 275

Number Theory. Topics covered will include the fundamental theorem of arithmetic, Euclid's algorithm, congruences, Diophantine problems, quadratic reciprocity, arithmetic functions, and distribution of primes. If time allows, we may also discuss some geometric methods, coming from lattice point counting, such as Gauss's circle problem and Dirichlet divisor problem, as well as some applications of Number Theory to coding theory and cryptography.

MATH 276

Algebraic Geometry. Topics include affine and projective varieties, the Nullstellensatz, rational maps and morphisms, birational geometry, tangent spaces, nonsingularity and intersection theory. Prerequisite: Math 271; recommended previous courses in Analysis, Galois Theory, Differential Geometry and Topology are helpful but not required; or permission of the instructor.

MATH 277

Mathematical Methods in Data Science. In this course, students will learn about common mathematical representations of data, the mathematical foundations of matrix factorization and tensor decomposition, and their application to many tasks in machine learning and data science. These decomposition techniques are integral tools in studying large-scale and multi-modal data and form the basis for many approaches to the topic modeling, dimension reduction, and clustering tasks. Potential topics include PCA, nonnegative matrix factorization, higher-order SVD, nonnegative tensor decompositions, K-means clustering, optimization techniques for these models, and applications in machine learning, data science, signal processing, and network science.

MATH 280

Introduction to Partial Differential Equations. Classifying PDEs, the method of characteristics, the heat equation, wave equation, and Laplace's equation, separation of variables, Fourier series and other orthogonal expansions, convergence of orthogonal expansions, well-posed problems, existence and uniqueness of solutions, maximum principles and energy methods, Sturm-Liouville theory, Fourier transform methods and Green's functions, Bessel functions. Prerequisites: Differential equations course and Math 231, or permission of instructor.

MATH 281

Dynamical Systems. The theory of continuous dynamical systems was developed largely in response to the reality

that most nonlinear differential equations lack exact analytic solutions. In addition to being of interest in their own right, such nonlinear equations arise naturally as mathematical models from many disciplines including biology, chemistry, physiology, ecology, physics, and engineering. This course is an introduction to and survey of characteristic behavior of such dynamical systems. Applications will be an integral part of the course with examples including mechanical vibrations, biological rhythms, circuits, insect outbreaks, and chemical oscillations. Prerequisite: Differential equations course and Math 231, or permission of instructor.

MATH 283

Mathematical Modeling. Introduction to the construction and interpretation of deterministic and stochastic models in the biological, social, and physical sciences, including simulation studies. Students are required to develop a model in an area of their interest.

MATH 285

Methods in Modern Modeling. Models are applied on a daily basis to provide insight into any number of current world problems. From diseases to government policy, modeling techniques are being used to predict outcomes and manage populations. With the advent of more computational power and data collection, novel model types and techniques for analysis are being derived. We will explore current models and techniques which are used across multiple disciplines. We will consider agent-based (or individual-based) modeling, and ordinary differential equation models with parameter estimation, along with additional topics. Students will have a chance to investigate using analytical and computational skills, that can be applied across a diversity of fields. Course has pre-requisites.

MATH 286

Stochastic Methods in Operations Research. Queuing theory, decision theory, discrete event simulation, inventory modeling, and Markov decision processes.

MATH 287

Deterministic Methods in Operations Research. Linear, integer, nonlinear, and dynamic programming. Applications to transportation problems, inventory analysis, classical optimization problems, and network analysis, including project planning and control.

MATH 288

Social Choice and Decision-Making. This course focuses on the modeling of individual and group decisions using techniques from game theory. Topics will include: basic concepts of game theory and social choice theory, representations of games, Nash equilibria, utility theory, non-cooperative games, cooperative games, voting games, paradoxes, impossibility theorems, Shapley value, power indices, fair division problems, and applications.

MATH 289

Special Topics in Mathematics. Topics vary from year to year and may include: algebraic geometry, algebraic topology, fluid dynamics, partial differential equations, games and gambling, Bayesian analysis, geometric group theory or other topics.

MATH 293

Mathematics Clinic. The Mathematics Clinic provides applied, real-world research experience. A team of 3-5 students will work on an open-ended research problem from an industrial partner, under the guidance of a faculty advisor. Problems involve a wide array of techniques from mathematical modeling as well as from engineering and computer science. Clinic projects generally address problems of sufficient magnitude and complexity that their analysis, solution and exposition require a significant team effort. Students are normally expected to enroll in Math 393 (Advanced Mathematics Clinic) in the subsequent semester. Prerequisite: permission of instructor.

MATH 294

Methods of Applied Mathematics. Applications of linear algebra and differential equations in modeling. Concepts such as vector spaces, solvability of linear systems, eigenvalues and eigenvectors, singular value decomposition, matrix norms, matrix exponentials, solution of ODEs, nonlinear dynamical systems, bifurcations, phase-plane analysis, stability of fixed points, and Floquet theory are reviewed. They are illustrated through examples such as Leslie population models, Google's PageRank algorithm, Markov chains, image compression, population dynamics, linear and nonlinear oscillators, predator-prey models, spread of epidemics, enzyme kinetics, gene/protein regulatory networks and the like. Familiarity with undergraduate level differential equations and linear algebra will be helpful.

MATH 306

Optimization. The course emphasizes nonlinear programming. It covers numerical methods for finite-dimensional optimization problems with fairly smooth functions. Both non-constrained and constrained optimizations will be discussed. Certain degree of emphasis will be given to the convergence analysis of the numerical methods. Prerequisite: multivariable calculus and numerical linear algebra.

MATH 334

Applied Analysis. The course integrates concepts from real and functional analysis with applications to differential equations. We will introduce theoretical concepts such as metric, Banach and Hilbert spaces as well as abstract methods such as the contraction mapping theorem, with a focus on applications rather than exhaustivity. We will apply these concepts to the study of integral equations, ordinary differential equations and partial differential equations. The main methods will be Fourier series and the Sturm-Liouville theory.

MATH 335

Integral Transforms and Applications. Transforms covered will include: Fourier, Laplace, Hilbert, Hankel, Mellin, Radon, and Z. The course will be relevant to mathematicians and enginners working in communications, signal and image processing, continuous and digital filters, wave propagation in fluids and solids, etc.

MATH 337

Real and Functional Analysis I. Abstract measures, Lebesgue measure on Rn, and Lebesgue-Stieljes measures on R. The Lebesgue integral and limit theorems. Product measures and the Fubini theorem. Additional related topics as time permits. Prerequisite: Math 231 and Math 232.

MATH 338

Real and Functional Analysis II. Continuation of Math 337. Some of the topics covered will be: Banach and Hilbert

spaces; Lp-spaces; complex measures and the Radon-Nikodym theorem. Prerequisite: Math 337.

MATH 351

Time Series Data Analysis. Analysis of time series data by means of particular models such as ARIMA. Spectral analysis. Associated methods of inference and applications. Prerequisite: permission of instructor.

MATH 352

Nonparametric Statistics. Treatment of statistical questions which do not depend on specific parametric models. Examples are testing for symmetry of a distribution and testing for equality of two distributors. Elementary combinatorial methods will play a major role in the course. Prerequisite: Math 252 or permission of instructor.

MATH 353

Asymptotic Methods in Statistics with Applications. Modes of convergence for random variables and their distributions; central limit theorems; laws of large numbers; statistical large sample theory of functions of sample moments, sample quantiles, rank statistics, and extreme order statistics; asymptotically efficient estimation and hypothesis testing. Prerequisite: Math 251 and 252; linear algebra; analysis (Math 231 and 232 or equivalent).

MATH 354

Reliability Theory. Structural properties and reliability of complex systems; classes of life distributions based on aging; maintenance and replacement models; availability, reliability, and mean time between failures for complex systems; Markov models for systems; elementary renewal theory. Prerequisite: Math 251. Math 256 would be helpful but not essential.

MATH 355

Linear Statistical Models. A discussion of linear statistical models in both the full and less-than-full rank cases, the Gauss-Markov theorem, and applications to regression analysis, analysis of variance, and analysis of covariance. Topics in design of experiments and multivariate analysis. Prerequisite: Linear algebra and a year course in probability and statistics.

MATH 357

Deterministic and Stochastic Control. The course consists of two parts. The first part is lecture-based, and will cover both deterministic and stochastic control. In deterministic control, we will cover the calculus of variations, Pontryagin's maximum principle, the linear regulator, and controllability. In stochastic control, we will review some stochastic analysis, and then will cover dynamic programming, viscosity solutions, the stochastic version of Pontryagin's maximum principle, and backward stochastic differential equations. The second part will be a seminar on applications of optimal control to engineering and to mathematical finance problems. The course will cover a new method based on Malliavin calculus to solve backward stochastic differential equations analytically, with potential new applications. Prerequisite: Math 256 (Stochastic Processes). Knowledge of Ito calculus is helpful but not required, and will be reviewed in the course.

MATH 358

Mathematical Finance: Fixed Income and Derivatives. This course starts by describing financial derivatives in the following markets: equities, fixed income, credit, commodities, currency. Emphasis is then on the pricing of these securities by formulae or numerical techniques (Monte Carlo, trees, finite differences). We analyze the following models: Black-Scholes, stochastic volatility, Vasicek, SOFR and others. Stochastic calculus is used extensively. Real options are also discussed. Prerequisite: Math 256 (Stochastic Processes) would be good preparation but is not required.

MATH 359

Computational Statistics. This course will cover standard methods typically discussed in a computational statistics course. Areas include optimization methods: Newton, quasi-Newton. Fisher scoring, iteratively reweighted least squares, EM algorithm. These methods presented for both settings: univariate and multivariate. Combinatorial optimization: Simulated Annealing and genetic algorithm. Simulation and Monte Carlo Integration methods: exact and approximation. MCMC methods: Metropolis-Hasting algorithm and Gibbs Sampling. R/R-studio will be used for all computational needs. Prerequisite: Math 251 or equivalent.

MATH 362

Numerical Methods for Differential Equations. This course is devoted to numerical integration of ordinary and partial differential equations. Methods such as Runge-Kutta and Adams formulas, predictor-corrector methods, stiff equation solvers and shooting method for BVPs are described. Other techniques based on interpolation via Lagrange and Chebyshev polynomials and cubic splines, numerical differentiation using finite differences, spectral and pseudo-spectral methods are introduced. Numerical methods for solving elliptic, parabolic and hyperbolic partial differential equations are presented including discussion of truncation error, consistency, stability, accuracy and convergence. Topics such as explicit vs. implicit schemes; implementation of Dirichlet, Neumann and Robin boundary conditions; operator splitting; Godunov methods for hyperbolic systems; direct and iterative methods for elliptic systems; Fourier and Chebyshev based spectral and pseudo-spectral methods are discussed. The Level-Set Method and its related PDEs may also be considered. A background in differential equations and some computational skills (e.g. MATLAB, Python or C) are assumed.

MATH 364

Machine Learning for Asset Pricing. Teaching a course on how to beat the market is an elusive endeavor. Techniques that work robustly (if they exist) are kept confidential. There has however been a spate of advances in deep learning that exploit the only fruitful concept that finance theory invented: arbitrage between different financial instruments. This class focuses on understanding these methods, when applied to simple instruments (stocks and bonds), whereas the mathematical finance class (MATH 358) covers financial options and stochastic calculus. The class is composed of 4 parts: optimization, machine learning, asset pricing theory, and deep learning for asset pricing. The weekly homework will be composed of a mix of mathematical questions, (short) computational problems, and more verbal questions, based on the simple book by Andrew Ang (Asset Management). Indeed, an important skill for an asset manager is the ability

to explain results to a lay audience. There will be an optional capstone project, where we will implement deep learning for asset pricing (following Pelger et al 2019). We will also cover more traditional methods, such as mean-variance, mean-variance with learning (Black-Litterman), linear factor models, linear programming, PCA, as well as some high-frequency trading. All models will be in discrete time, so no stochastic calculus is required. Prerequisites: linear algebra and probability.

MATH 365

Statistical Methods In Molecular Biology. Topics include statistical analysis of microarray data (the biological problem, modern microarray technology, high-dimensional small sample size data problem, key elements in a good experiment design, identifying sources of variation and data preparation, normalization techniques, statistical methods and algorithms for error detection and correction, statistical analysis workflow, software tools including R and bioconductor, and computational implementation of workflow methods and algorithms in R) and microarray data interpretation (use of linear models for analysis and assessment of differential expression, gene selection (Empirical Bayes, volcano plots) and gene classification (clustering, heatmaps)). Prerequisite: competency in scientific computing, linear algebra, an upper division course in statistics, familiarity with basic biology, access to a computer on which the Elluminate software can be run (requires latest JAVA runtime environment).

MATH 366

Data Mining. Data mining is the process for discovering patterns in large data sets using techniques from mathematics, computer science and statistics, with applications ranging from biology and neuroscience to history and economics. Students will learn advanced data mining techniques that are commonly used in practice, including linear classifiers, support vector machines, clustering, dimension reduction, transductive learning and topic modeling.

MATH 368

Advanced Matrix Analysis and Computations. This course is an in-depth study of advanced applied linear algebra and matrix computations. Topics covered will include: unitary, normal and symmetric matrices; Jordan canonical form; the minimal polynomial and the companion matrix; canonical forms; matrix factorizations; trace and determinant; positivity and absolute value; tensor product, block matrices and partial ordering, kernel functions, projections and positivity preserving mappings; matrix calculus(matrix exponent, matrix square root, log of matrix, sign of matrix, derivatives), matrix convexity; matrix means and inequalities, properties of positive and nonnegative matrices, primitive matrices, stochastic and doubly stochastic matrices; mean transformation; singular values. It will also cover many of the fundamental algorithms such as LU decomposition, Jacobi, Gauss-Seidel and SOR iterations, Krylov subspace methods (e.g., Conjugate Gradient and GMRES), QR and SVD factorization of matrices, eigenvalue problems via power, inverse, and Arnoldi iterations. The course is designed for those who wish to use applied matrix analysis and computations in their own research. A background in linear algebra and some computational skills (e.g., MATLAB, Python or C) are assumed.

MATH 375

Quantum Computing and Applications. While quantum computers are not in our homes yet, companies like Alibaba, Intel, IBM, Microsoft and DWave are developing them. We will cover in this class an introduction to quantum mechanics, quantum circuits, the quantum Fourier transform and applications, quantum search algorithms, quantum cryptography and quantum annealing. All the basic number theory necessary to understand Shor's algorithm will be provided in the course. Students will have the choice between a theoretical option, where they will gain a full understanding of Shor's algorithm, and a practical application to integer programming. We will discuss some possible applications: will Bitcoin disappear with the advent of quantum computing? Since quantum bits cannot be copied, and thus stolen, will quantum money become the standard of electronic commerce?

MATH 381

Mathematical Models in Fluid Dynamics. This course introduces the students to mathematical modeling of fluid flow and transport phenomena. We start with vectors, tensors, index notation, and derivation of the Navier-Stokes equations that govern fluid flow. We then use these to model a wide range of problems. Depending on the students' interests, topics may be chosen from: hydrostatics, minimal surfaces, soap films, and static shapes of sessile and hanging drops; shape oscillations of liquid drops in microgravity; pulsatile flow in compliant tubes as a model for blood flow in the cardiovascular system; slender-body theory and swimming of bacteria with flagella; linear acoustics and sound propagation; oscillations of gas bubbles, cavitation, and acoustics of bubbly liquids; dynamics of polymer molecules in shear flows as models of non-Newtonian fluids; dynamics of magnetic suspensions (ferrofluids); lubrication theory, thin liquid films and coating flows; thermo-capillary migration of drops and bubbles; flow in porous media, ground-water flow and Darcy's law; flow of rivers and estuaries; shallow water waves; geophysical fluid dynamics and climate modeling. Prerequisites: Vector Calculus, Differential Equations, Linear Algebra.

MATH 382

Perturbation and Asymptotic Analysis. Non-dimensionalization and scaling, regular and singular perturbation problems, asymptotic expansions; asymptotic evaluation of integrals with Laplace's approximation, Watson's lemma, steepest descents and stationary phase; perturbation methods in ordinary and partial differential equations; boundary layers and matched asymptotic expansions; method of multiple time scales; homogenization; WKB method, rays and geometrical optics. Prerequisite: differential equations.

MATH 384

Advanced Partial Differential Equations. Advanced topics in the study of linear and nonlinear partial differential equations. Topics may include the theory of distributions; Hilbert spaces; conservation laws, characteristics and entropy methods; fixed point theory; critical point theory; the calculus of variations and numerical methods. Applications to fluid mechanics,

mathematical physics, mathematical biology and related fields. Prerequisite: Math 280; Math 232 recommended.

MATH 385

Mathematical Modeling in Biology. With examples selected from a wide range of topics in biology and physiology, this course introduces both discrete and continuous bio-mathematical modeling, including deterministic and stochastic approaches. Methods for simplifying and analyzing the resulting models are also described. These include non-dimensionalization and scaling, perturbation methods, analysis of stability and bifurcations, and numerical simulations. The selection of topics will vary from year to year but may include: enzyme kinetics, transport across cell membranes, the Hodgkin-Huxley model of excitable cells, gene regulatory networks and systems biology, cardio-vascular, respiratory and renal systems, population dynamics, predator-prey systems, population genetics, epidemics and spread of infectious diseases, cell cycle modeling, circadian rhythms, glucose-insulin kinetics, bio-molecular switches, pattern formation, morphogenesis, etc. Prerequisite: advanced calculus, linear algebra, differential equations, probability theory, and basic numerical methods.

MATH 386

Image Processing. This will be a course on mathematical models for image processing and analysis. It will explore fundamental concepts such as image formation, image representation, image quantization, point operations, change of contrast, image enhancement, noise, blur, image degradation, edge and contour detection (such as the Canny edge detector), corner detection, filtering, denoising, morphology, image transforms, Fourier shape descriptors, image restoration, image segmentation, and applications. Time permitting, we will explore one or more concepts in computer vision, e.g. motion analysis, 3D shape reconstruction, or feature detection and tracking. All theoretical concepts will be accompanied by computer exercises, to be performed either in Matlab or in Python. All students will be required to work in small teams on a final project of their choice.

MATH 387

Discrete Mathematical Modeling. Discrete mathematics deals with countable quantities. The techniques used for discrete models often differ significantly from those used for continuous models. This course explores some of the main techniques and problems that arise in discrete mathematical modeling. Topics include combinatorial analysis, Markov chains, graph theory, optimization, algorithmic behavior and phase transitions in random combinatorics. The goal is for students to acquire sufficient skills to solve real-world problems requiring discrete mathematical models. Prerequisite: Probability and linear algebra. A previous course in discrete mathematics would be helpful.

MATH 388

Continuous Mathematical Modeling. A course aimed at the construction, simplification, analysis and interpretation of mathematical models, primarily in the form of partial differential equations, arising in the physical and biomedical sciences. Derivation and methods of solution: method of characteristics, separation of variables, Fourier and Laplace transforms. Examples such as traffic flow, steady and transient heat conduction, potential flow, advection-diffusion processes, wave propagation and acoustics. Dimensional analysis and scaling, perturbation theory, and bifurcation analysis. Students will normally work on a modeling project as part of the course. Familiarity with vector calculus, complex variables and differential equations will be helpful. Prerequisite: permission of instructor; Math 294 recommended.

MATH 389

Advanced Topics in Mathematics. Topics will vary from year to year.

MATH 393

Advanced Mathematics Clinic. Normally a continuation of Math 293. The Mathematics Clinic provides applied, real-world research experience. A team of 3-5 students will work on an open-ended research problem from an industrial partner, under the guidance of a faculty advisor. Problems involve a wide array of techniques from mathematical modeling as well as from engineering and computer science. Clinic projects generally address problems of sufficient magnitude and complexity that their analysis, solution and exposition require a significant team effort. Prerequisite: permission of instructor.

MATH 398

Independent Study (Master's Students). Directed research or reading with individual faculty.

MATH 451

Statistical Mechanics and Lattice Models. An intermediate-level graduate statistical mechanics course emphasizing fundamental techniques in mathematical physics. Topics include: random walks, lattice models, asymptotic/thermodynamic limit, critical phenomena, transfer matrix, duality, polymer model, mean field, variational method, renormalization group, finite-size scaling. Prerequisite: Math 251 or equivalent. The course will assume a basic familiarity with thermodynamics, as well as undergraduate-level real analysis and linear algebra.

MATH 452

Large-Scale Inference. This is a course for graduate students in statistics, mathematics and other fields that require an advanced background and deeper understanding of the theory and methods of high-dimensional data analysis. Bayesian hierarchical models and Empirical Bayesian approaches will be presented, as well as theory and application of multiple hypothesis testing such as false discovery rate. Applications will be presented on important and high-throughput medical problems particularly in genomic medicine.

MATH 454

Statistical Learning. This course is targeted at statisticians and financial engineering practitioners who wish to use cutting-edge statistical learning techniques to analyze their data. The main goal is to provide a toolset to deal with vast and complex data that have emerged in fields ranging from biology to finance to marketing to astrophysics in the past twenty years. The course presents some of the most important modeling and prediction techniques, along with

relevant applications. Topics include principal component analysis, linear regression, classification, resampling methods, shrinkage approaches, tree-based methods, clustering, and Bayesian MCMC modeling.

MATH 458

Quantitative Risk Management. This course focuses on developing tools to quantify and manage the different sources of risk in financial markets. In particular, we will develop tools to forecast volatility, calculate different measures of risk such as Value-at-Risk and Expected Shortfall by Monte Carlo (with or without copulae) or other methods, such as quadratic VaR. We will also study coherent measures of risk, risk aggregation, and capital allocation. Special attention will be given to credit risk: pricing risky bonds, counterparty risk, credit default swaps, securitization, and consumer credit risk. Other types of risk impacting financial institutions, such as model risk, operational risk, and liquidity risk, will also be covered. Topics will vary according to current market needs and may include ESG, climate, and cryptocurrency derivatives. Prerequisite: Math 256.

MATH 461

Level-Set Methods. This course provides an introduction to level-set methods and dynamic implicit surfaces for describing moving fronts and interfaces in a variety of settings. Mathematical topics include: construction of signed distance functions; the level-set equation; Hamilton-Jacobi equations; motion of a surface normal to itself; re-initialization; extrapolation in the normal direction; and the particle level-set method. Applications will include image processing and computer vision, image restoration, de-noising and de-blurring, image segmentation, surface reconstruction from unorganized data, one- and two-phase fluid dynamics (both compressible and incompressible), solid/fluid structure interaction, computer graphics simulation of fluids (i.e. smoke, water), heat flow, and Stefan problems. Appropriate for students in applied and computational mathematics, computer graphics, science, or engineering. Prerequisite: advanced calculus, numerical methods, computer programming.

MATH 462

Mathematics of Machine Learning. Machine learning is a rapidly growing field that is concerned with finding patterns in data. It is responsible for tremendous advances in technology, from personalized product recommendations to speech recognition in cell phones. This course material covers theoretical foundations, algorithms, and methodologies for machine learning, emphasizing the role of probability and optimization and exploring a variety of real-world applications. Through lecture examples and programming projects, students will learn how to apply powerful machine learning techniques to new problems. Students are expected to have a solid foundation in calculus and linear algebra as well as exposure to the basic tools of logic and probability, and should be familiar with at least one modern, high-level programming language

MATH 466

Advanced Big Data Analysis. This graduate level course is designed to give students a snapshot of recent techniques used to analyze, statistically and algorithmically, extremely large datasets. To accomplish this goal, the course will start with an applied and quick introduction to necessary optimization background. From there we will introduce students to topics such as spectral graph clustering, fast kernel methods, compressed sensing, among others. We will highlight applications of these methods to diverse areas such as genomics and recommender systems, but the bedrock of the course will be theory. To that end, students are expected to have a solid foundation in probability and analysis, as well as comfort with algorithmic thinking.

MATH 498

Independent Research (PhD students). Directed research or reading with individual faculty.

MATH 499

Doctoral Study.

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